OFFICERS AND COUNCIL.

CONTENTS OF PROCEEDINGS.

NAME INDEX (ABSTRACTS).

XXIII. *The Hatchet Planimeter.* By F. W. Hill[*].

THE hatchet planimeter consists essentially of a tracing-point and a convex chisel-edge rigidly connected, the point and the edge being in the same plane. When the point is moved along any line, the edge describes a curve of pursuit.

The object of this paper is to investigate how the instrument may be used to determine areas.

Let the tracer start from a point O inside the area, move along any line to the perimeter, then round the perimeter and back along the same line to O ; the solution of the problem consists in finding an expression for the angle AOB between the initial and final positions OA, OB of the line joining the tracing-point and chisel-edge.

All attempts to express the area of the curve in terms of this angle proved futile except in a few special cases, such as the circle and square; but the magnitude of the angle may be found in the form of an infinite series, the most important term of which is a multiple of the area. The complexity of the result would seem to show that no simple geometrical explanation is possible.

Let the tracer move a distance r along a straight line (fig. 1) ; then, if χ, χ' be the initial and final inclinations of the rod to the line, c the length of the rod, it is easily proved that

$$\tan\frac{\chi'}{2} = e^{\frac{r}{c}}\tan\frac{\chi}{2}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let the area of the curve be divided into infinitesimal triangles having their vertices at a point O inside the curve,

Fig. 1.
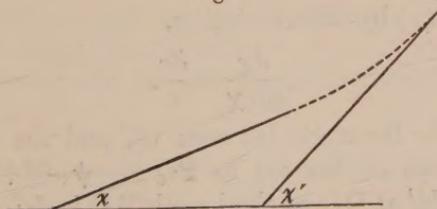
and for their bases elements of the perimeter, the problem is

* Read June 22, 1894.

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best attacked by making the tracer move round each of these triangles in turn, travelling along every radius-vector twice, in opposite directions. Taking any triangle OPQ (fig. 2),

Fig. 2.
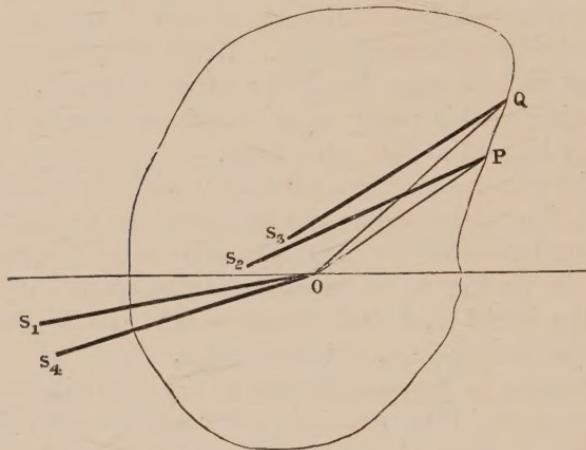
let θ, $\theta + d\theta$ be the inclinations of OP, OQ to a fixed line OX; ϕ, ϕ', $\phi' + d\phi'$, $\phi + d\phi$, the inclinations of the rod to the fixed line in the positions S_1O, S_2P, S_3Q, S_4O respectively; r, $r + dr$ the lengths of OP, OQ.

Then by (1)

$$\tan \frac{\theta - \phi'}{2} = e^{-\frac{r}{c}} \tan \frac{\theta - \phi}{2} \quad \cdots \quad (2)$$

and

$$\tan \frac{\theta + d\theta - \phi' - d\phi'}{2} = e^{-\frac{r + dr}{c}} \tan \frac{\theta + d\theta - \phi - d\phi}{2}$$

for the motions from O to P, and from Q to O; hence

$$\frac{d\theta - d\phi'}{\sin (\theta - \phi')} = -\frac{dr}{c} + \frac{d\theta - d\phi}{\sin (\theta - \phi)} \cdot \quad \cdots \quad (3)$$

Also from (1) by differentiating,

$$\frac{d\chi}{\sin \chi} = \frac{dr}{c}.$$

Now if ψ be the angle between OP and the tangent PQ, the inclinations of the rod to PQ are $\psi - \theta + \phi'$ at P, and $\psi - \theta + \phi' + d\phi'$ at Q, and the length P Q is ds;

therefore

$$\frac{d\phi'}{\sin (\psi - \theta + \phi')} = \frac{ds}{c},$$

or

$$cd\phi' = \{\sin\psi\cos(\theta-\phi') - \cos\psi\sin(\theta-\phi')\}ds$$
$$= rd\theta\cos(\theta-\phi') + dr\sin(\theta-\phi').$$

Substituting this value of $d\phi'$ in (3),

$$\frac{cd\theta - rd\theta\cos(\theta-\phi')}{\sin(\theta-\phi')} - dr = -dr + \frac{c(d\theta-d\phi)}{\sin(\theta-\phi)},$$

or

$$\frac{d\theta-d\phi}{\sin(\theta-\phi)} = \frac{d\theta}{2}\left(1-\frac{r}{c}\right)\cot\frac{\theta-\phi'}{2} + \frac{d\theta}{2}\left(1+\frac{r}{c}\right)\tan\frac{\theta-\phi'}{2}$$

$$= \frac{d\theta}{2}\left(1-\frac{r}{c}\right)e^{\frac{r}{c}}\cot\frac{\theta-\phi}{2} + \frac{d\theta}{2}\left(1+\frac{r}{c}\right)e^{-\frac{r}{c}}\tan\frac{\theta-\phi}{2}$$

by (2);

$$\therefore\ d\theta-d\phi = \left[\left(1-\frac{r}{c}\right)e^{\frac{r}{c}}\cos^2\frac{\theta-\phi}{2} + \left(1+\frac{r}{c}\right)e^{-\frac{r}{c}}\sin^2\frac{\theta-\phi}{2}\right]d\theta$$

$$= \tfrac{1}{2}\left[\left(1-\frac{r}{c}\right)e^{\frac{r}{c}} + \left(1+\frac{r}{c}\right)e^{-\frac{r}{c}}\right]d\theta$$

$$-\tfrac{1}{2}\left[\left(1+\frac{r}{c}\right)e^{-\frac{r}{c}} - \left(1-\frac{r}{c}\right)e^{\frac{r}{c}}\right]d\theta\cos(\theta-\phi)$$

$$= \left(1 - \frac{r^2}{2c^2} - \frac{r^4}{8c^4} - \frac{r^6}{144c^6} - \dots\right)d\theta$$

$$-\left(\frac{r^3}{3c^3} + \frac{r^5}{30c^5} + \dots\right)\cos(\theta-\phi)d\theta.$$

Or

$$c^2d\phi = \frac{r^2d\theta}{2} + \frac{r^4}{8c^2}d\theta + \frac{r^6}{144c^4}d\theta$$

$$+ \left(\frac{r^3}{3c} + \frac{r^5}{30c^3} + \dots\right)d\theta\cos(\theta-\phi). \quad . \quad . \quad (4)$$

This equation gives the change in the direction of the rod when the tracing-point has moved round an elementary triangle and integrating round the curve there results, for the angle between the initial and final positions of the rod, the expression

$$c^2\Phi = A + \frac{1}{8c^2}\int r^4 d\theta + \frac{1}{144c^4}\int r^6 d\theta + \dots$$

$$+ \int\left(\frac{r^3}{3c} + \frac{r^5}{30c^3} + \dots\right)\cos(\theta-\phi)d\theta$$

R 2

where A is the area of the curve, and the integrations extend from $\theta = 0$ to $\theta = 2\pi$. Thus if the greatest breadth of the curve is less than the length of the planimeter, the most important term in the expression is A, and it remains to estimate the effect of the other terms.

First, considering those terms which depend only on the position of O,

$$\frac{1}{8c^2} \int r^4 d\theta = \frac{Ak^2}{2c^2},$$

where Ak^2 is the moment of inertia of the area about the polar axis through O. This term is therefore least when O is the mass-centre of the area.

$$\frac{1}{144c^4} \int r^6 d\theta \text{ is less than } \frac{2A}{144}\left(\frac{a}{c}\right)^4,$$

where a is the greatest radius-vector from O, and, if $\dfrac{a}{c}$ is as great as $\dfrac{1}{8}$, is less than one per cent. of the area.

Of the terms depending on ϕ the most important is

$$\frac{1}{3c} \int r^3 \cos(\theta - \phi) d\theta.$$

In these small terms the value of $c^2\phi$ at any point may be taken to be the area of that part of the curve already described by the tracing-point, and the terms can be evaluated when the equation of the curve is known. If, however, the area of the curve is less than that of a square of side c, so that the greatest value of ϕ is less than 1, it is easy to approximate.

$$\frac{1}{3c} \int r^3 \cos(\theta - \phi) d\phi = \frac{1}{3c} \int r^3 \cos\theta d\theta + \frac{1}{3c} \int r^3 \phi \sin\theta d\theta$$

$$- \frac{1}{6c} \int r^3 \phi^2 \cos\theta d\theta + \ldots$$

Now, the axis of x being the initial position of the rod,

$$\frac{1}{3c} \int r^3 \cos\theta d\theta = \frac{1}{c} \int x dA = \frac{A\bar{x}}{c},$$

where $\bar{x}$ is the abscissa of the mass-centre of the area. The other terms are multiplied by $\left(\dfrac{a}{c}\right)^3$, and being integrals of oscillating quantities are very small.

Thus

$$c^2\Phi = A + \frac{Ak^2}{2c^2} + \frac{A\bar{x}}{c} + \frac{P}{c^3},$$

where P is very small.

If O is the mass-centre of the curve k^2 is a minimum and $\bar{x}=0$. In this case

$$c^2\Phi = A\left(1 + \frac{\rho^2}{2c^2}\right)$$

very nearly, where $A\rho^2$ is the moment of inertia about the polar axis at the mass-centre.

If O is not near the mass-centre, let the tracing-point be moved again round the perimeter, following the same path as before, but with the initial direction of the instrument exactly reversed ; the effect of this is to change the sign of c and to give

$$c^2\Phi' = A + \frac{Ak^2}{2c^2} - \frac{A\bar{x}}{c} - \frac{P}{c^3};$$

hence

$$\frac{c^2}{2}(\Phi + \Phi') = A + \frac{Ak}{2c^2}$$

$c\Phi$ being the distance between two indentations may perhaps be measured to within $\frac{1}{100}$ inch, so that there will be an error in $c^2\Phi$ of at least $\frac{1}{20}$ sq. inch, if c is 10 inches ; if, therefore, $\dfrac{Ak^2}{2c^2}$ is less than $\frac{1}{20}$ sq. inch, it may be neglected.

Of curves with the same greatest breadth, the circle has the greatest area, and $\dfrac{Ak^2}{2c^2}$ is less than $\frac{1}{20}$ sq. inch for a circle of radius $1\frac{1}{2}$ inches.

Finally, then, if O be near the mass-centre, and the greatest breadth be less than 3 inches, the formula $c^2\Phi = A$ will give a result as accurate as the nature of the instrument will allow ; for greater areas the value of $\dfrac{Ak^2}{2c^2}$ must be estimated and taken into account. Also it is easily verified that if Φ is less than $\frac{3}{13}$, or the area less than 23 sq. inches, it is sufficient to measure the chord of the arc $c\Phi$ instead of the arc.

DISCUSSION.

Prof. HENRICI did not agree with the statement that the instrument was necessarily inaccurate, and thought geometry might aid analysis to find the proper starting point. For a symmetrical curve he had shown that a point existed, starting from which the area traced by the hatchet end was zero.

Dr. MACFARLANE GRAY thought Prof. Henrici's latter argument was vitiated by his figures not being correctly drawn ; but this Prof. Henrici disputed.

Mr. O. G. JONES said Prof. Henrici's construction was not obvious, for the cusp curves traced by the hatchet end depended on the starting point.

Mr. YULE suggested that by shortening the planimeter it might be possible to bring the third and second terms of Mr. Hill's formula into greater prominence, and by going round the curve more than once determine the first and second moments.

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XXIV. *The Influence of Temperature on the Specific Heat of Aniline. By* E. H. GRIFFITHS, *M.A.**

[Plates VIII. & IX.]

OUR knowledge of the effect of changes of temperature upon the specific heat of substances is limited. The reason of this is evident, for, in addition to the difficulties of thermo-metry, the experimental methods usually adopted are based on comparisons in which water is used as a standard, and our knowledge of its capacity for heat at different tempe-ratures is far from satisfactory. The conclusions of Rowland (1879), of Bartoli and Stracciati (1893), and my own investigation completed in 1892, all point to the fact that the specific heat of water diminishes as the temperature rises to 20° C., at which temperature Bartoli and Stracciati find it is a minimum. My observations did not extend beyond 27° C., up to which temperature I found no signs of a minimum,

* Read October 26, 1894.

Ext.Temp.N.P.   Wt. of Aniline = 358·20 , stirring 300 rev. per. min.
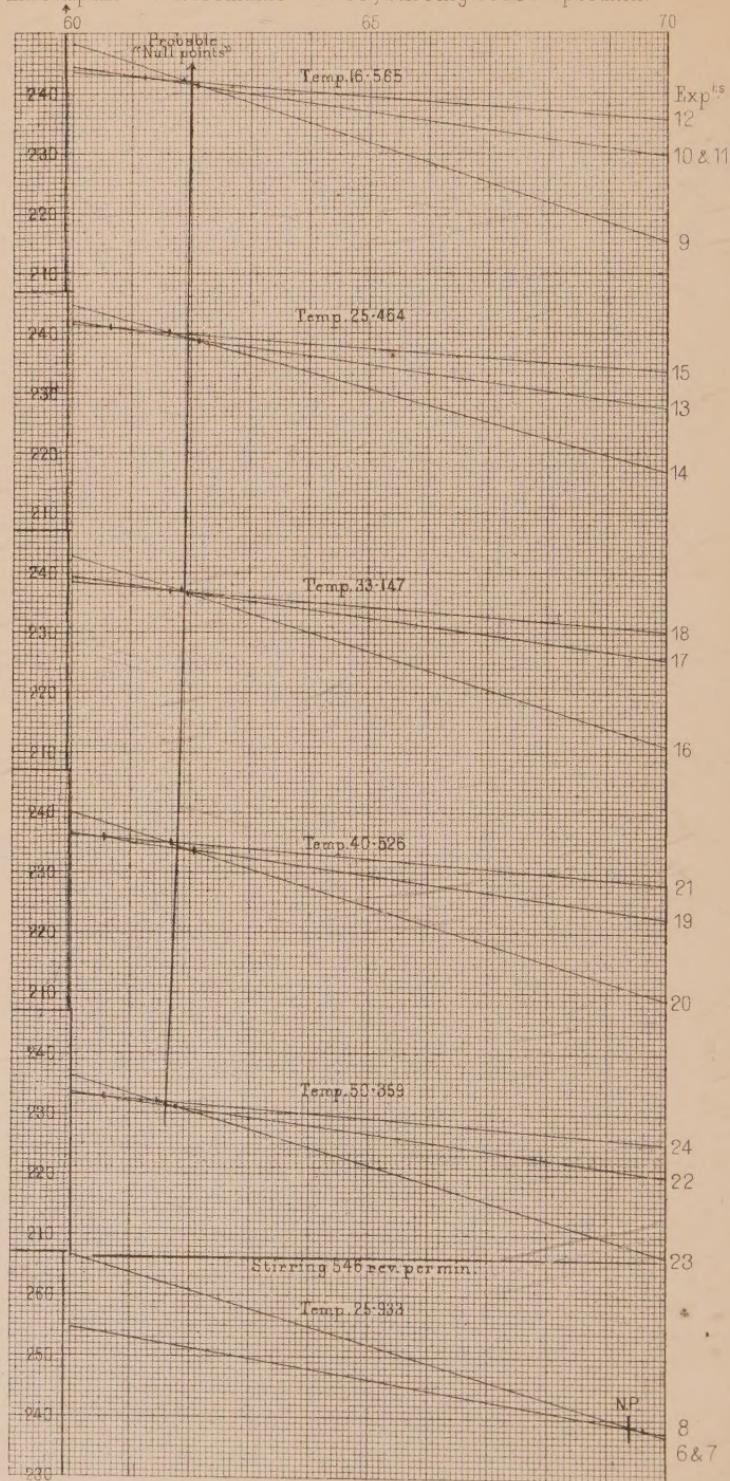
Ext.Temp.N.P.   Wt. of Aniline = 569·53, stirring 300 rev. per min.
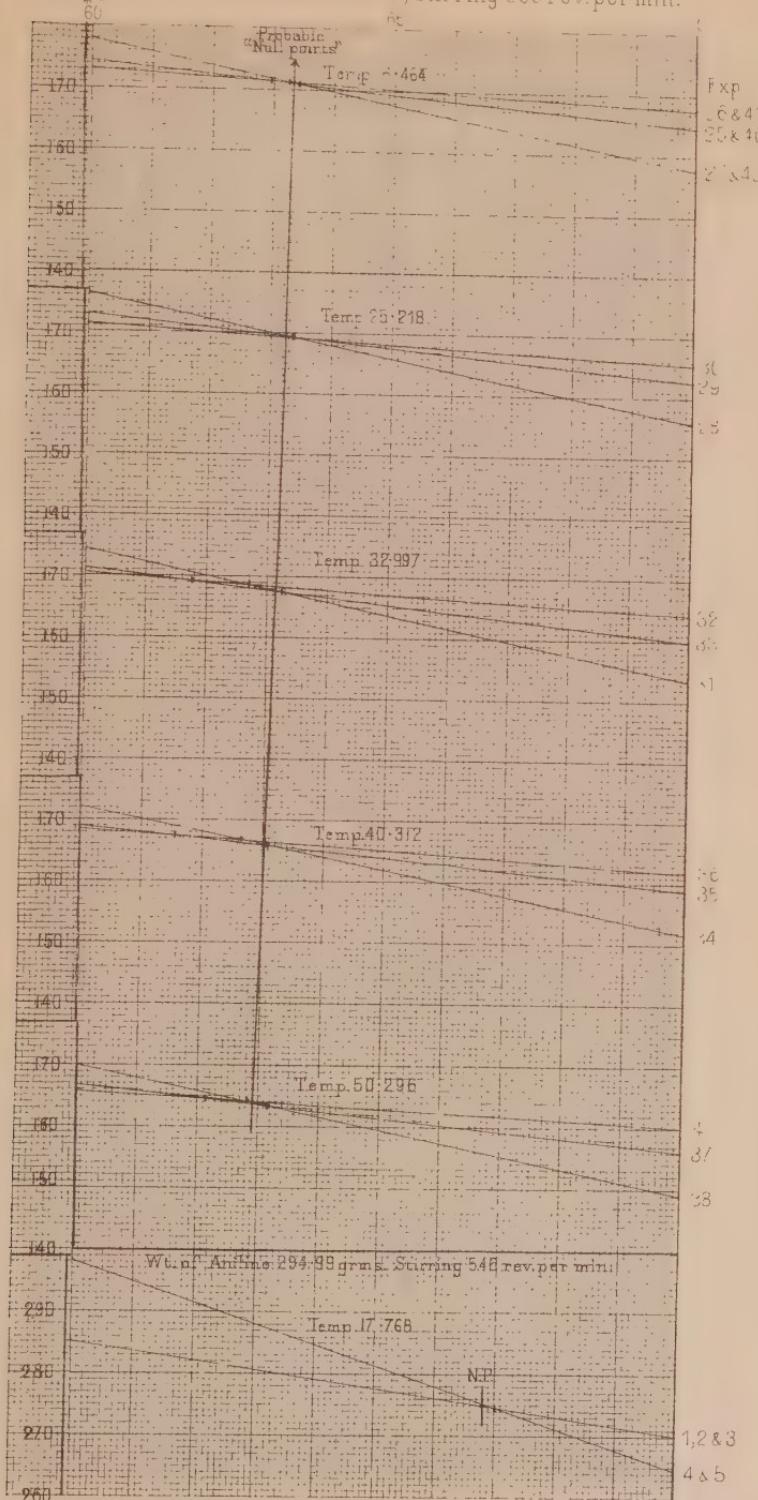
Chart A.
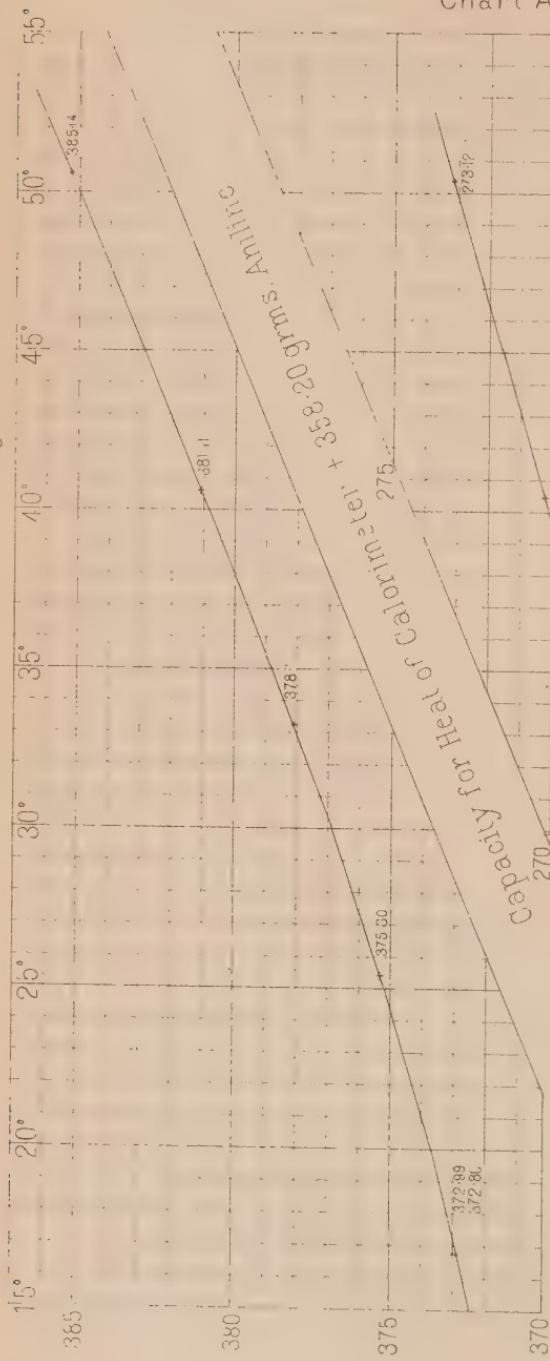
Chart B

515°

5300

50° &94

45° 6949

40° 2.44

35 5221

30 5196

25 5176

5165

5.47

5157

5300

5250

5200

5150

Water Equivalent of Calorimeter

83 60

83 38

82 88

82 19

81 50

80 83

80 49

80 08

80 01

79 82

84

83

82

81

80
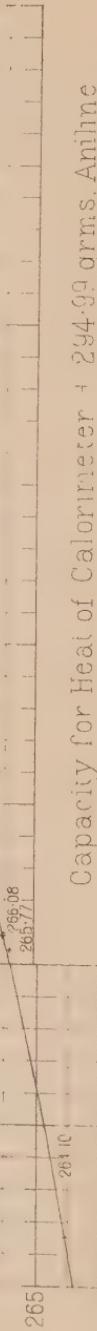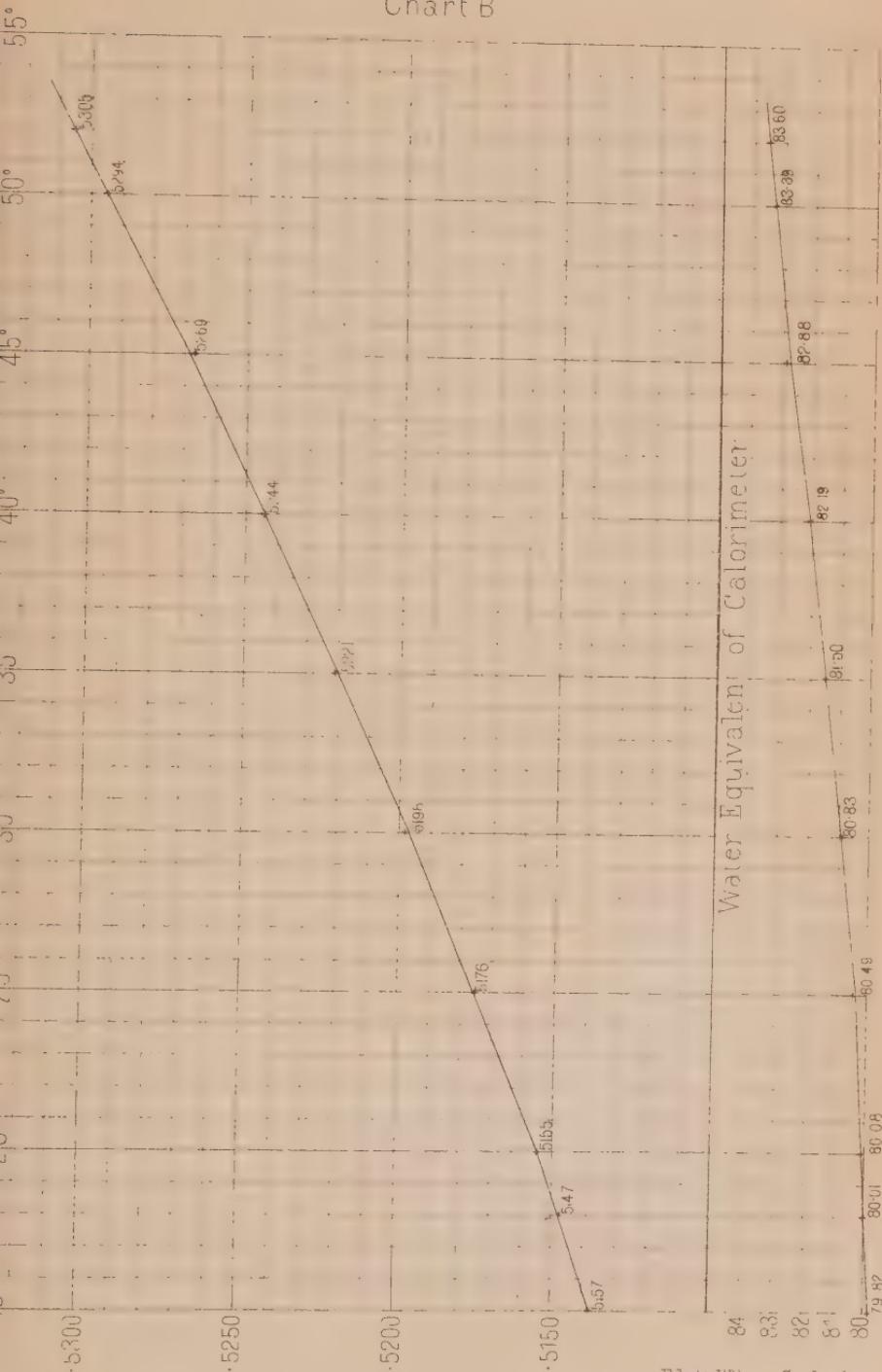
which Rowland places at about 34° C. When such discrepancies exist with regard to the standard, it is not surprising that the conclusions arrived at regarding other substances are unsatisfactory.

For other reasons water is by no means an ideal standard for calorimetry. Its capacity for heat is so great that the changes in temperature caused by the immersion in it of bodies whose specific heats are small are too minute to be measured with exactness under ordinary circumstances. The difficulty is surmounted in practice by causing the immersed body to cool through a considerable range. Other errors are, however, introduced by this method, for the transference of the hot body into the cool water can rarely be conducted without loss of heat. What we require is a liquid of small capacity for heat whose temperature-coefficient of specific heat is accurately known. It should also, if possible, be a liquid easily obtainable in a fairly pure state and should have a low vapour-pressure at ordinary temperatures. I believe that in Aniline we possess such a liquid, and I hope to be able to adduce sufficient evidence to lead to the conclusion that its specific heat is now known with considerable accuracy over a range of 15° to 52° C.

The method I have adopted is an old one, viz. a supply of heat to the interior of the calorimeter by means of an immersed conductor whose ends are kept at a constant potential-difference. I admit that there are difficulties inseparable from the method (such as doubts as to the actual resistance of the conductor when its temperature is raised by the current, &c.), but when once these difficulties are overcome, there is no such accurate means of determining the quantity of heat passed into a calorimeter in a given time.

The limits of this paper forbid any detailed account of the somewhat complicated apparatus used in this investigation, and I do not contend that it is of the form which I should have selected had my object simply been to determine the specific heat of aniline. I am at present engaged in a determination of the latent heat of evaporation of water and other liquids at different temperatures, consequently the apparatus has been designed and put together in anticipation only of that investigation, and many portions of it are unnecessary and in fact

detrimental to the inquiry I am now describing. Hence I have been compelled to adopt methods of observation which may not, at first sight, seem the most advantageous.

I have in my possession an apparatus for maintaining the walls of a chamber at a constant temperature. The arrangement has been fully described in a communication entitled "The Mechanical Equivalent of Heat," Phil. Trans. clxxxiv. A (1893), and as I shall somewhat frequently have to refer to this paper, I will now label it with the letter " J." Briefly, the apparatus as there described consists of a tank containing about 20 gallons of water within which is a steel chamber shaped somewhat like a hat-box with vertical sides, the space between the double walls and floor containing rather more than 70 lbs. of mercury, which communicates by a narrow tube with a gas-regulator differing but little from the ordinary pattern. Thus a row of about 50 tiny gas-jets (placed under a tube through which water is always flowing) are so controlled as to be distinctly affected by a change of $\frac{1}{400}$° C. in the temperature of the steel chamber. An addition has been made to the apparatus since the publication of the description in paper J, where it is stated that tap-water continually passed into the tank through a silver tube placed above the gas-jets : this plan answered admirably from the temperature of the tap-water (10° to 12° C.) to about 30° C. Last year, however, Prof. Callendar and I wished to use the tank for purposes of comparison of platinum and air thermometers up to 50° or 60° C., when it was found that alterations in temperature presented themselves, occasionally amounting to as much as $\frac{1}{50}$° C.,—the lag in the temperature of the large mass of mercury being so great that when the gas was lowered by the action of the regulator, the resulting inflow of cold water tended to lower the temperature of the tank before the reaction of the regulator could again heat the inflow. As now arranged, when working above temperatures about 20° C. a small motor acts as a heart and, the tap-water being shut off, pumps the tank-water itself round through the silver tube placed above the gas-jets. The water, by passing through the pump &c., is slightly cooled : thus the work of the regulator is confined to . simply supplying the heat lost by convection, radiation, &c., and it performs this task admirably. As an illustration, I

may mention that in the series of over 50 experiments treated of in this communication on only one occasion did the temperature of the steel chamber change by as much as $\frac{1}{100}$° C. throughout the duration of an experiment. On the solitary occasion that a change amounting to nearly $\frac{1}{50}$° C. was observed, the cause was found in the caking of the lime through which the gas was passed on its way to the regulator, and, in consequence, the experiment was discarded before working out its results.

A large screw, placed in the tank-water, revolved at about 800 times per minute and raised a considerable sea—the flow passing round, under, and over the steel chamber, the top of which was about 4 in. beneath the surface of the water.

The calorimeter was formed of brass and was suspended by glass tubes which, after passing through the lid of the steel chamber and through the water, projected above the lid of the tank. The diameter and depth were each about 10 centims. and the capacity about 700 cubic centims. Within it was suspended a silver flask with which three silver tubes communicated. One was connected with a glass tube passing to the exterior through which substances could be introduced. The second tube, which was about 18 feet in length, was, after leaving the top of the flask, twisted into a spiral within the calorimeter ; the other end, terminating in the lid of the calorimeter, was then connected by a glass tube with a four-way tap entirely immersed in the outer tank. The third tube, which opened into the bottom of the flask, communicated with about 30 feet of copper tubing placed in the tank-water. The object of the whole arrangement was that the gas on leaving the calorimeter (after evaporating any liquid in the flask) should have acquired the temperature of the calorimeter, and that any gas passed into the flask should assume the temperature of the outer tank. True, this portion of the apparatus was not necessary to the experiments on aniline, but I have felt it advisable to describe it, as explaining some of the subsequent operations.

A coil of fine german-silver wire, supported on glass pillars, was placed within the calorimeter and so arranged as to surround the silver flask. Communicating with this coil were four leads—Nos. 1 and 2 of which were fastened to one

end, and Nos. 3 and 4 to the other end of the coil at the roof
of the calorimeter, and, passing up through the steel vessel,
communicated with the exterior. Nos. 1 and 3 formed the
ends of a circuit in which were placed a high-resistance
galvanometer and the Clark cells ; Nos. 2 and 4 a circuit
which contained a rheochord of special construction (for a
description see paper J), reversing-keys, and storage-cells.
It was thus possible to maintain the ends of the calorimeter-
coil at a difference of potential equal to that of any number
of Clark cells ; and I am convinced that the variations in the
potential-difference during the course of an experiment rarely
amounted to $\dfrac{E}{10,000}$. (The grounds on which this somewhat
bold statement is based are fully given in paper J, p. 283.)
The contents of the calorimeter could thus be heated or cooled
without any disturbing effects from external causes. By
means of the electrical arrangements above described, the
supply of heat could be regulated at will and accurately de-
termined, while, by the insertion into the flask of a volatile
liquid such as ether, it was possible, by adjusting the current
of dried air, to regulate the rate of cooling.

Particulars of the Clark cells have already been published
(paper J, pp. 286–288), and the whole 36 maintain to-day
almost exactly the same relative values as they had when
compared with the Cavendish standard in 1892 : there is,
therefore, every reason to believe that their absolute value
remains unchanged. I have not yet had time to repeat the com-
parison with the Cavendish standard, but I hope to do so shortly.
I do not, however, anticipate that any correction will be found
necessary. The whole of these cells were contained in a bath,
through which the tap-water was turned by means of a regulator
whenever the temperature exceeded 15° C.

The calorimeter previously described differed considerably
from that used in my determinations of the mechanical equi-
valent. During these experiments the air-pressure in the
space between the walls of the calorimeter and those of the
steel chamber containing it was reduced to from ·5 to ·2 millim.
as measured by McLeod's gauge; and my observations proved
that there was a strangely rapid diminution in the loss by
convection, &c., when the pressure fell below half a milli-

metre[*].   I proposed, however, during the experiments for which this second calorimeter was designed to maintain the calorimeter and the steel chamber at nearly equal temperatures, and it did not, therefore, appear so necessary to guard against convection and radiation gains and losses.   Previous experience had convinced me of the absolute necessity for keeping this intramural space dry, for I found that the slightest moisture in the contained air has the most astonishing effect in changing the conditions.   It was therefore necessary that all joins should be absolutely tight, for the lid of the steel chamber was under water, and I proposed, in the preliminary experiments, to place water within the calorimeter.   The use of indiarubber was forbidden, as I intended to insert ether in the silver flask and aniline in the calorimeter ; and even if such had not been the case, indiarubber connexions are, at the best, unsatisfactory and unreliable. I decided, therefore, not to commence my experiments until I found that the apparatus was absolutely gas-tight in all parts ; and I may mention that the greater part of last summer was unsuccessfully devoted to the endeavour to secure perfection in this respect.   During last winter I spent considerable time in the effort to obtain some suitable medium by which to join glass to metal ; and with the assistance of Mr. Thomas I was at last successful in procuring an alloy by which an air-tight join could be formed[†], and which would stand considerable changes of temperature.   Five glass tubes passed from the calorimeter to the steel lid, rendering ten such glass-to-metal joins necessary, besides several similar ones in the exterior connexions.

In the spring of this year the intramural space was exhausted until the reading of the McLeod gauge connected therewith was reduced to 11, indicating a pressure of about 0·12 millim.   The apparatus was then left untouched for a month except that the temperature was occasionally raised or lowered, and at the end of that time the reading of the gauge was still less than 12.   Dry air was then readmitted to this

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[*] A similar conclusion was arrived at by Bottomley ; see Phil. Trans. 1887, A.

[†] Proc. Camb. Phil. Soc. 1894.

space, and the silver flask with its connected tubes (embracing about 50 feet of tubing with several joins) tested in a similar manner. Those who have had to deal with low pressures will understand that when all was found satisfactory a great difficulty had been surmounted. I did not retain this vacuum during the experiments, as I felt that it would subject the glass tubes &c. to a continuous strain which the conditions of the experiments rendered unnecessary. The labour had not been lost, however, for I was able to count with confidence on the gas-tightness of the whole apparatus.

I now pass to a description of that vital part of any such investigation, viz. the thermometry. The method I proposed to adopt necessitated extremely accurate measurements of small differences of temperature. The actual elevation was of little consequence, and therefore the use of differential thermometers immediately suggested itself. Two platinum thermometers (labelled AB and CD) were constructed with great care: four stout platinum leads passed down the stem of each, supported and insulated in the usual manner by small disks of mica, and the resistance of all these leads was made as equal as possible before attaching the coils. Great attention was given to this matter, and it is safe to assume that the leads in no case differed amongst themselves by 1 in 10,000. The coils, consisting of a particularly pure sample of platinum wire, were then attached, and several days were devoted to securing their equality. Their resistance in ice was about 18 ohms : thus $\frac{1}{1800}$ of their resistance could be directly determined in the box. The galvanometer swing was about 500 for a change of ·01 in the box ; and such equality was secured that when both thermometers were placed in ice (the necessary precautions being taken with regard to exterior connexions &c.), no difference in the swing of the galvanometer could be observed ; thus they differed by a quantity certainly less than 1 in 100,000. This equality, although not a necessity, was a great convenience.

Although cut from the same length of wire and insulated in a precisely similar manner, the coils did not possess exactly the same coefficients. The resistances in steam and sulphur were repeatedly determined and checked by observations in the vapour of aniline. Both thermometers were on several

occasions heated to a red heat, the hard glass tubes containing them becoming slightly bent in the process ; but since this annealing no further change has been observable in them. The method of completely standardizing such instruments has been fully described by Professor Callendar and myself in Phil. Trans. 1891, A, and I need not therefore here dwell upon it. The values of δ differed slightly, viz. 1·513 and 1·511 ; but such a difference, even if not allowed for, would over the range 0° to 100° C. in no case cause an error exceeding about $\frac{1}{2000}$° C. in elevation. These thermometers were so connected that the compensating leads of AB were placed in series with the coil of CD, and *vice versâ*. Any heating of the stem of AB or CD, therefore, added an equal resistance to each arm of the bridge ; and as the leads were everywhere bound together, the indications were absolutely independent of all changes in temperature except those of the bulbs.

Great care is required to accurately determine the fixed points of such thermometers. For example, a difference of 1° at 100° C. made a difference of ·067835 ohm: thus a difference of 1 millim. on the barometer caused a difference of ·00244 ohm. Readings of the pressure were taken on a standardized $\frac{3}{4}$-inch diameter mercury-barometer, and also on a compensated sulphuric barometer suspended beside it. It was found that the observations, if corrected to 100° C. by the pressure as deduced from a mercury-barometer, were not in close agreement ; but if by the sulphuric-acid barometer, the results were satisfactory. On examination it appeared that the first observations of each series, when corrected for pressure, were in harmony whichever barometer was used ; but such was not the case with subsequent ones of the same series. The cause is, I believe, to be found in the necessary handling of the barometer that takes place when adjusting the mercury to the ivory point. The temperature within the case undergoes a change, and the indications of the thermometer usually placed within the metal tube surrounding the barometer give but little information as to the real temperature of the mercury column, whereas the sulphuric-acid barometer requires no manipulation and is independent of such temperature changes. As an example of the consistency of the boiling-

point when corrected by the H_2SO_4 barometer, I give the following *consecutive* results :—

(Difference in R for 1° C. at 100°=·067835.)

Date.	R after correction for temperature of box, plug errors, &c.	Barometer corrected to sea-level, lat. 45° and temp. 0° C.	Boiling-point.	Correction to 100° C.	R_1 (at 100° C.).
Thermometer AB.					
June 3 ...	24·58798	761·10	100·040	−·00271	24·58527
„ 5 ...	24·57187	754·67	99·802	+·01342	24·58529
July 7 ...	24·58336	759·25	99·972	+·00190	24·58526
Thermometer CD.					
June 3 ...	24·58606	761·10	100·040	−·00271	24·58335
„ 5 ...	24·56961	754·52	99·796	+·01383	24·58344
July 7 ...	24·58163	759·34	99·975	+·00168	24·58333

The above examples will show that it is possible to determine the resistance at 100° C. to better than 1 part in 250,000, and nearly the same order of accuracy is obtainable, with proper precautions, in the vapour of sulphur.

I believe the most difficult point to accurately determine is freezing-point, and for really satisfactory determinations it is advisable to use distilled-water ice ; but if this is manufactured in the usual manner there is great danger that some trace of salt may be carried from the freezing-mixture to the ice. One precaution I have found advantageous when using ordinary ice, viz., on adding water to the powdered ice it is advisable to use water resulting from the melting of that ice itself rather than either distilled or tap water.

I will not burden this paper with any further detailed account of the standardization of these thermometers. Between March 4th and July 7th the fixed points were determined on over 30 occasions, in ice, steam, or sulphur vapour, and no variation which would affect temperature measurements between 0° and 100° C. by as much as $\frac{1}{2000}$° C. presented itself

during the latter half of these observations, the record and reductions of which alone make a large pile of manuscript.

The two thermometers, with their leads connected as described, were placed at opposite ends of a bridge wire of platinum-silver. During the spring of this year this wire was subjected to a most careful calibration by what was practically Carey Foster's method, and it proved to be more unequal than I had expected. It was therefore re-calibrated by a different method in which a resistance-box was used as a shunt, and the agreement between the results was satisfactory. The whole wire was 80 centim. long and had a total resistance of about ·4 ohm. For convenience, and to avoid thermal effects, a similar wire connected with the galvanometer was laid alongside it, and the sliding-piece was fitted with a screw so arranged that a small turn of the screw-head made contact with both wires *.

The wire and contact-maker were covered by a thick copper shield (the screw-head projecting through a narrow slit) passing from end to end of the bridge: thus the temperature of the wire was kept uniform. By means of a vernier the divisions on the scale could be read to $\frac{1}{10}$ millim., which with this wire and thermometers AB and CD indicated at 50° C. a temperature-difference of ·000915° C.† The temperature-coefficient of this wire was found to be ·00029. The resistances of the different parts of the wire were, after correction for the errors of individual coils, &c., merely expressed in terms of the mean box ohm, the absolute value being of no consequence so long as the fixed points were determined in terms of the same standard. The remaining two arms of the bridge were constructed of german-silver. They were wound together, boiled in paraffin, placed in a bottle, and I expended much care in finally adjusting them until equal. Their resistance was about 5 ohms and the galvanometer about 8 ohms, which, assuming the resistance of the thermometers as about 20 ohms each, would give nearly the maximum of sensitiveness. A single storage-cell was always

* This method of making the connexion is due to Professor Callendar, and is exceedingly effective and convenient.

† A thicker wire giving a more open scale was tried at first, but found to be less convenient than the one finally adopted and calibrated.

used, and a resistance of 40 ohms was placed in the battery circuit when the thermometers were in ice. A table was then calculated which gave the resistance necessary in the battery circuit when the thermometers were at any temperature in order that C^2R (where R is the thermometer resistance) should be constant. Thus the rise in the temperature of the thermometer coils due to current-heating was always the same, and consequent errors were eliminated, a point to which I attach considerable importance.

The value of $R_1 - R_0$ in thermometer AB was 6·88815 ; therefore a difference of 1 ohm at 50° C. indicated a difference of 14°·5177 C., and as

$$\frac{d\,pt}{dt} = \left\{ 1 - \delta\,\frac{2t - 100}{100^2} \right\}^*,$$

the degree value of any bridge-reading at other temperatures can be deduced.

There was no difficulty, in the arrangement above described, in reading with certainty a difference of $\frac{1}{1000}$° C., and, as an illustration, I may mention the fact that if the thermometers were placed in separate hypsometers side by side on the bench and one of the hypsometers was then removed to the ground (about 3 feet below), the difference in the bridge-wire reading thus caused slightly exceeded ·4 millim.

Before leaving this portion of the subject I may mention that I carefully compared the thermometer AB with two standards from the International Bureau of Weights and Measures, which passed through my hands this summer on their way to Sydney University, and further with a separate standard sent me by the Bureau in February 1893. As the particulars of this comparison are now in the press †, I need not refer to them further than to say that the results were most satisfactory.

When the experiments were in progress the thermometer CD was placed in a hole drilled in the wall of the steel chamber parallel to the side and separated from the inner

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* Phil. Trans. 1891, A. p. 142.
† 'Science Progress,' September 1894

surface by only $\frac{1}{16}$ in.: thus it assumed the temperature of the walls surrounding the calorimeter. Thermometer AB passed within the calorimeter and was immersed to an inch above its bulb in the contained liquid. The reading on the bridge-wire thus gave the difference between the temperature of the calorimeter and the walls of the surrounding chamber.

To ascertain the actual temperature and the *constancy* of the temperature of the surrounding walls, a second hole, like that already described, was drilled in the steel, in which a mercury thermometer of open scale was placed. The stem, where it protruded above the lid of the tank, was surrounded by a glass tube up which the motor pumped the tank-water, which, passing out at the top, returned to the tank. The stem, therefore, was maintained at a steady temperature, and the fluctuations observed might be considered as solely due to changes in the walls of the steel chamber. The stem of thermometer A (used at the lower temperatures) was divided into millimetres, about 27 to a degree, and No. II. (used at the higher) about 20 millim. per degree. They were observed through a telescope containing a micrometer-scale giving direct readings to $\frac{1}{10}$ of a millimetre. The tenth of one of these divisions could be estimated, and thus changes of $\frac{1}{1000}°$ C. (*i. e.* about ·025 millim.) were easily observable. This was important, as the bridge-wire readings had to be corrected for any movement in the temperature of thermometer CD—that is, the temperature of the walls of the chamber.

The stirrer (which consisted of two nearly vertical narrow paddles, reaching from top to bottom of the calorimeter) was so placed as to throw the liquid across the silver flask and coil against the thermometer. This form of stirrer was not the one I should have adopted had the apparatus been designed solely for the experiments which I am now describing. In that case I should have preferred the form described in paper J, which threw the liquid from the bottom to the top of the calorimeter ; and I believe that such irregularities as have presented themselves in these experiments, especially with the smallest mass, are due to insufficient stirring. When the calorimeter is full of liquid the nature of the stirring is of less consequence ; and as I proposed in the experiments for

which the apparatus was designed to completely fill the calori-
meter, I adopted the form which it appeared to me would
generate the least heat compatible with efficient mixing. The
stirrer was supported in the same manner as that described in
a former paper ; that is, its bearings were entirely external to
the calorimeter except at the base, where, to check vibration,
an agate cylinder hung within a ring *.

In place of the 2000 revolutions per minute used in my
former work I confined myself, for the reasons above stated,
to a comparatively slow rotation of from 500 to 300 revolutions
per minute. The revolving shaft was electrically connected
with the chronograph in such a manner that the time of
every 1000 revolutions was automatically recorded on the
tape.

Before commencing the observations on aniline I introduced
299·35 grms. (*in vacuo*) of water into the calorimeter, and,
with a view to ascertaining the water-equivalent, performed
a few experiments to determine the rate of rise when a known
current was passed through the coil. The point was not of
great importance, as I proposed to determine the specific heat
of aniline by a method requiring no knowledge of the water-
equivalent, which, however, when the specific heat of aniline
was known, could be deduced from the experiments, and thus
a previous determination by means of water would serve as
an independent test of the accuracy of the results. I had not
covered the german-silver coil with any insulator, since,
aniline being itself an insulator, the precaution was un-
necessary ; and this freedom to use the naked wire was one
of the chief reasons which led to the adoption of aniline as a
suitable liquid for my subsequent work. I was therefore
aware that I could not expect very satisfactory results from
the use of water. The polarization was considerable, as shown
by the back E.M.F. at the termination of an experiment, and
also by the difficulty experienced in making any accurate
determination of the coil-resistance. However, it was sub-
sequently found that the experiments gave very fair results.

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* A portion of the shaft of the stirrer was constructed of ivory, to .
diminish its thermal conductivity.

The temperature was raised from about ·5° C. below to ·5° C. above the surrounding temperature (*i. e.*, through about 12 centim. of the bridge-wire), and the times of passing the several divisions of the bridge-wire were recorded on the chronograph.   Five experiments were performed, and the results of the first three gave a water-equivalent of 80·1, and Nos. 4 and 5 of 79·8.   They were thus divided into groups before the results were calculated, as the coil showed decided signs of change in resistance between the two batches of experiments.   I had proposed to continue these experiments with a different mass of water, but became alarmed as to the effect on the coil of the change that was going on, and decided to withdraw the water, for had the coil become useless from any cause, the whole apparatus would have had to be taken to pieces and the base of the calorimeter unsoldered, &c., involving a delay of at least a week or two.

During the time that the water was in the calorimeter I performed a considerable number of experiments to determine the rise in temperature due to the work done by the stirrer.   With this form of stirrer and the low rates, tr^2 was practically a constant, where t was the time of rising 1 centim. of the bridge-wire and r the rate of revolution per second. The resulting correction was comparatively small, amounting to about $\frac{1}{25}$ of the heat generated by the current during the water-equivalent experiments ; thus any small error in the correction introduced when eliminating the heat supplied by the stirrer became unimportant.

After the water was withdrawn, the whole apparatus was kept for three days at a temperature between 40° and 50° C., while dried air was forced continuously through every part of it.   At the end of this time a sulphuric-acid bulb (through which the air passed on its exit from the apparatus) showed no increase in weight.   I then introduced 294·99 grms. (*in vacuo*) of aniline into the calorimeter *.

I will at once state that no great pains were taken to secure an absolutely pure specimen of aniline.   For my subsequent

* The precautions observed in order that the mass introduced might be accurately determined are fully described in paper J, p. 418.

experiments the purity of the aniline was a matter of no consequence, as I only required to know the actual capacity of the calorimeter and contents at different temperatures.

Again, as regards the purposes of this investigation I think that it will be of greater service if I am able to supply experimenters with the specific heat of a quality of aniline at all times procurable, rather than if I gave the constants of a quality only to be obtained with difficulty. The sample I used was supplied by Messrs. Harrington Bros., as "pure colourless," and had but a light brown tinge. It was redistilled by me once before use, and I also determined its boiling-point, which was in agreement with that of a specially pure sample which I had examined on a previous occasion *. Further (and this I have always found to be a fair test of the purity), the temperature of the boiling-point did not change during distillation.

From the time that aniline was first introduced into the calorimeter all signs of polarization disappeared, and the insulation of the coil was very perfect.

I will now describe the method of conducting an experiment.

The tank was first brought to the desired temperature, and the experiment was not proceeded with until the tank-temperature had had time to become steady. Freshly distilled ether was then introduced into the silver flask, and dried air passed through it (the stirrer in the surrounding aniline being continually at work) until the *whole* of the ether was evaporated. The taps connected with the silver flask were then closed, the key inserted in the storage-circuit, the connexion with the Clark cells made, and the rheochord adjusted until the potential-difference at the ends of the coil was shown by the high-resistance galvanometer to be equal to that of the Clark cells used. Three or four of these cells were placed in parallel arc, and the files thus formed in series : therefore, when using an electromotive force of 4 Clark cells, we had really in use 12 or 16.

* Trans. 1891 A, p. 64.

Throughout an experiment the attention of my assistant *
was directed to keeping the potential balance as even as
possible. I meanwhile had to observe and mark the time of
transit across the graduations of the bridge-wire. A reversing-
key was maintained at a constant period of oscillation (about
twice per second), and as the temperature rose the oscillations
of a dead-beat galvanometer mirror (which were viewed
through a telescope fitted with a micrometer-scale) steadily
diminished, and the moment when they ceased could, to my
surprise, be determined with great accuracy. By pressing a
key, the time was recorded on the chronograph tape.

The mercury-thermometer inserted in the walls of the steel
chamber was observed at regular intervals, any alteration
noted, and a correction afterwards applied to the bridge-wire
reading—the value of each division of the bridge-wire in
terms of a millimetre of the thermometer-scale being known.
The time of each 1000 revolutions of the stirrer was auto-
matically recorded on the tape : thus the only notes that had
to be taken during an experiment were of the changes of the
mercury-thermometer. The chronograph was one of some-
what novel construction, for which I am indebted to Mr. E.
A. Pochin. It was worked by a water motor, had a triple set
of recording hammers, and was controlled by an electric
clock which was compared at regular intervals with a " rated "
Dent's chronometer, but as the greatest gain in rate observed
was less than 1 in 12,000, no correction was necessary.

I find it impossible to give a detailed record of the obser-
vations, as it would fill a volume. I therefore propose to give
an example in full of one of the experiments by drawing at
random one of the leaves from the mass of records. (Table I.
p. 250.)

* I take this opportunity of returning my thanks to Mr. C. Green,
Scholar of Sidney College, Cambridge, for his able assistance during
these experiments.

TABLE I.

Aniline Exp. No. 26 : 4 cells.   July 28th.

Cooled to reading 36 cm. b.w.   Temp. of b.w. = 67°·5 F.
Temp. Clark Cells = 15°·27 C. throughout.

Experiment commenced at 1–17–30.

Bridge-wire reading in centims.	Time by chrono-graph.	Number of revolutions.	Time.	Time per 1000 revo-lutions.	Temp. by thermo-meter Á.
	secs.		secs.	secs.	millim.
54·5	331·4				166·53
55·0	347·7	0	348·1		
55·5	366·9				
56·0	384·4				
56·5	403·4			204·5	166·55
Mean	366·8				
61·5	582·5	1000	552·6		166·58
62·0	600·5				
62·5	619·4				
63·0	638·1			204·6	
63·5	656·0				166·60
Mean	619·3				
68·0	819·9	2000	757·2		166·61
68·5	838·5				
69·0	856·2			204·2	
69·5	874·9				
70·0	892·6	3000	961·4		166·60
Mean	856·4				
75·0	1079·6			205·3	166·60
75·6	1098·7				
76·0	1118·3				
76·5	1137·4				
77·0	1156·2	4000	1166·7		166·59
Mean	1118·0				

Mean reading of thermometer A = 166·58 millim. = 16°·459 C.

Reduction of the Results.

Let θ_0 be the temperature of the surrounding envelope and θ_1 that of the calorimeter at any time t. Let the change in temperature per second due to the work done by the stirrer be σ, and let ρ equal the gain in temperature per second due to the combined effects of convection, conduction, and radiation when the difference in temperature between the calorimeter and the surrounding envelope is unity.

Let a wire whose resistance is R at θ_0 and R_1 at θ_1 be immersed in a liquid contained in the calorimeter, and let the mass of the contained liquid be M and its specific heat S_1 when the temperature is θ_1.

Let the capacity for heat of the calorimeter be w_1 when the temperature is θ_1.

Thus the thermal capacity of the calorimeter and contents at any temperature θ_1 is $S_1 M + w_1$.

If the ends of the wire be kept at a constant potential-difference of E, then we get (paper J, p. 367) :—

$$\frac{E^2}{J \cdot R_1(S_1 M + w_1)} + \{\dot{\sigma} - \rho(\theta_1 - \theta_0)\} = \frac{d\theta_1}{dt}.$$

Since σ is small and we can make $\theta_1 - \theta_0$ of any magnitude we please, it is always possible to obtain a value of θ_1 such that $\sigma - \rho(\theta_1 - \theta_0) = 0$; and in that case

$$\frac{E^2}{J \cdot R_1(S_1 M + w_1)} = \left(\frac{d\theta_1}{dt}\right)_e,$$

the suffix denoting the nature of the supply.

This temperature (viz. that value of θ_1 which causes $\sigma - \rho(\theta_1 - \theta_0)$ to vanish) I shall denote throughout this paper by the term "null point," by which I mean that value of θ_1 at which the observed rate of rise is due to the electrical supply only, and I shall use θ_N to indicate that temperature.

Two or three methods of finding θ_N suggest themselves. If the stirrer be set working when there is no electrical supply, then θ_1 will gradually approximate to θ_N, and a few observations as to the rate of change in θ_1 will supply sufficient information to enable the observer to set the value near to

θ_N, and the calorimeter can then be left to gradually assume the true value. I have given this method a fair trial, for the apparatus has been left working for twenty-four hours at a time, and the value of θ_N ascertained with precision.

For example, at the rate of 9·10 revolutions per second it was found that the value of $\theta_N - \theta_0$ (expressed in terms of the mean bridge-wire centimetre) was about 9·2 (*i. e.* 0°·85 C. approximately), and that when the rate was 5·00 the value was about 2·7 ($=0°·25$ C. approximately); and it is noticeable that the ratio 9·2 : 2·7 is nearly that of $(9·1)^2 : (5·0)^2$, thus indicating that the heat developed by the stirrer varied approximately as r^2, where r is the rate of revolution.

A modification of this method renders it possible to determine the thermal value of the stirring work. After obtaining θ_N as above, find what value of θ_1 will cause the temperature to remain steady when the heating effect is that due to the stirring and a potential-difference of one Clark cell. Call this temperature θ_{Ne} ; we have thus two equations,

$$\sigma - \rho(\theta_N - \theta_0) = 0,$$

and

$$\sigma + \frac{e^2}{J \cdot R_1(S_1 M + w_1)} - \rho(\theta_{Ne} - \theta_0) = 0 ;$$

hence

$$\frac{\sigma}{\sigma + \dfrac{e^2}{J \cdot R_1(S_1 M + w_1)}} = \frac{\theta_N - \theta_0}{\theta_{Ne} - \theta_0}.$$

Whatever method is adopted the experiments involve much labour and time, for a redetermination of θ_N must be made for each change in θ_0, as it is probable that owing to changes in the viscosity and surface-friction of the contained liquid, θ_N would be some function of θ_0 when the rate of revolution of the stirrer is constant ; and a complete determination of the value of θ_N for each value of θ_0 would have doubled or trebled the length of the investigation.

It has always appeared to me to be possible that the value of ρ is a function of the rate of change in the temperature of the calorimeter. The temperature-gradient from the calorimeter to the surrounding walls must alter slightly with the rate of change in θ_1, and the rate of loss or gain in temperature of the calorimeter must as a consequence be affected ; or, in

other words, I think it probable that the value of θ_N is a function of $\dfrac{d\theta_1}{dt}$, hence the value of θ_N obtained by experiments such as those described above, where the temperature of the calorimeter is constant, may differ appreciably from the value of θ_N when the temperature is rising.

The method I finally adopted must diminish, although it does not entirely eliminate, any error due to this change in θ_N.

Let several experiments be performed similar in all respects except that the potential-difference is changed in each case.

Let the potential-difference of a Clark cell be e, and let n be the number of cells used.

Then

$$\frac{(ne)^2}{JR_1(S_1M + w_1)} + \{\sigma - \rho(\theta_1 - \theta_0)\} = \frac{d\theta_1}{dt};$$

hence

$$\left(\frac{d\theta_1}{dt}\right) \times \frac{1}{n^2} = \frac{e^2}{JR_1(S_1M + w_1)} + \frac{\sigma - \rho(\theta_1 - \theta_0)}{n^2}.$$

Let us assume for the time that θ_N is independent of n, then, when $\theta_1 = \theta_N$, we get

$$\frac{d\theta_N}{dt} \times \frac{1}{n^2} = \frac{e^2}{JR_1(S_1M + w_1)};$$

hence

$$\frac{d\theta_N}{dt} \times \frac{1}{n_1^2} = \frac{d\theta_N}{dt} \times \frac{1}{n_2^2} = \frac{d\theta_N}{dt} \times \frac{1}{n_3^2} = \&c.$$

where n_1, n_2, n_3 &c. are the number of cells used in each case.

If, therefore, we plot the curves (which in this case are straight lines since the variables, viz., $\rho(\theta_1 - \theta_0)$ and R_1, S_1, and w_1 may for small changes of temperature be considered as linear functions of θ_1, see paper J, pp. 442–448), which we obtain by taking $\dfrac{d\theta_1}{dt} \times \dfrac{1}{n^2}$ as ordinate, and θ_1 as abscissa, they will intersect where the abscissa is θ_N, and there only; and since the value of $\dfrac{d\theta_1}{dt}$ can be experimentally determined for different values of θ_1, the observations themselves can

be made to give the value of θ_N, and we can thus find $\dfrac{e^2}{J.R_1(S_1M + w_1)}$, the rise due to the electrical supply alone.

A small deviation from the true value of θ_N is of little consequence, for the resulting value of $-\dfrac{d\theta_1}{dt} \times \dfrac{1}{n^2}$ will only differ from $\dfrac{d\theta_N}{dt} \times \dfrac{1}{n^2}$ by the quantity $\dfrac{\rho(\theta_1 - \theta_N)}{n^2}$, and since ρ is small as compared with the other magnitudes, $\theta_1 - \theta_N$ must be considerable before the error becomes appreciable.

As this is an important point, I will select one of the three-cell experiments (No. 37) in order to show the relative magnitude of the various quantities, and the probable limit of error arising from the assumption that θ_N is independent of n. The differences in temperature are expressed in terms of the temperature difference equivalent to 1 centim. of the bridge-wire, but would of course have the same relative values in any scale.

$$\rho = \cdot000082.$$
$$\sigma = \cdot000230.$$
$$\left(\frac{d\theta_N}{dt}\right) = \cdot014670.$$

Let us assume that the value taken for θ_N differed from the true value by as much as 1 bridge-wire degree (as will be seen when the experimental results are given, this is an excessive estimate), then $\theta_1 - \theta_N = 1$, hence (*supra*) the value of $\dfrac{d\theta_1}{dt}$ would differ from

$$\frac{d\theta_N}{dt} \text{ by } \frac{\rho(\theta_1 - \theta_n)}{n^2} = \frac{\cdot000082 \times 1}{9} = \cdot000009.$$

When the values of $\dfrac{d\theta_1}{dt} = \dfrac{1}{n^2}$ (where $n = 2, 3,$ or 4) were plotted as ordinates with θ_1 as abscissa, the resulting straight lines were found to intersect not in one but in three points, which, however, were so near to each other that (except in one case) the probable error introduced by assuming that any one intersection gave the true value of θ_N was less than that above indicated. The triangle formed by joining the three points of intersection was almost invariably of one form and of nearly equal dimensions, indicating that its existence was

not due to experimental irregularities but was connected in
some way with the difference in the rate of rise with different
values of n. I think that my impression that ρ and hence
θ_N are functions of $\frac{d\theta_1}{dt}$ is thus confirmed.

In any case the error caused by the assumption that θ_N is
the same with different rates of increase in temperature, must
be less than the error resulting from the assumption that it is
the same when the temperature is rising and when it is
steady.

Again, the null point finally selected for each group of ex-
periments was always obtained in the same manner from the
three points of intersection. Any error, therefore, is of the
same nature in each case, and as the specific heat was obtained
by the subtraction of one ordinate from another, the effect of
any error is diminished considerably. True, when the absolute
value of the ordinate is used in order to obtain the water
equivalent of the calorimeter, any error in the position of θ_N,
and therefore in the length of the ordinate, would produce
its full effect, and might be appreciable, but as regards the
determination of the specific heat by the method of differences
it is almost certainly negligible. It would be represented by
$\frac{(\rho_1 - \rho_2)(\theta_1 - \theta_N)}{n^2}$, where ρ_1 and ρ_2 are the values of ρ when
the masses of the liquid are M_1 and M_2; and since

$$\rho_1(M_1 S_1 + w_1) = \rho_2(M_2 S_1 + w_1),$$

the limit of error is given by the expression

$$\frac{\rho_1 S_1(M_2 - M_1)}{M_2 S_1 + w_1} \times \frac{\theta_1 - \theta_N}{n^2}.$$

Now the value of $\frac{S_1(M_2 - M_1)}{M_2 S_1 + w_1}$ in these experiments was

about $\frac{1}{4}$, hence we get $\frac{\rho_1}{4} \cdot \frac{\theta_1 - \theta_N}{n^2}$ as the expression for the

error, and this would in no case amount to $\frac{1}{1000}$ of $\frac{d\theta_N}{dt}$.

I assume, therefore, that the values of $\frac{d\theta_N}{dt}$ given by the
experiments are sufficiently approximate.

If C_b is the value, in degrees C. of the air-thermometer, of the temperature difference equivalent to 1 centim. of the bridge-wire*, then

$$\frac{d\theta_N}{dt} \times \frac{R_1 \times C_b}{e^2}$$

gives the rise per second in degrees C. with unit resistance and unit potential-difference.

The reciprocal of the quantity thus found gives us T, the time of rising 1°, when $E=1$ and $R=1$.

Now

$$\frac{T}{J} = \sigma_1 M + w_1.$$

Let the value of T be T_1 when M is M_1, and T_2 when M is M_2.

Then we get

$$S_1 M_1 + w_1 = \frac{T_1}{J},$$

and

$$S_1 M_2 + w_1 = \frac{T_2}{J},$$

hence

$$S_1 = \frac{T_1 - T_2}{J(M_1 - M_2)}.$$

We can now find the value of $S_1 M_1$, and therefore

$$w_1 = \frac{T_1}{J} - S_1 M_1.$$

Thus both the specific heat of the liquid and the water-equivalent can be found if two groups of experiments with different masses of liquid have been performed at the same temperature.

Since $J = 4\cdot198 \times 10^7$† ergs, when the unit of heat is "the quantity of heat required to raise 1 grm. of water through 1° C. of the air-thermometer at 15° C.," it follows that the results obtained at different temperatures are all expressed in terms of that unit, and are not dependent upon our knowledge of the changes in the specific heat of water.

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* See p. 243, *supra*.
† Proc. Roy. Soc. lv. p. 26.

Corrections.

Before proceeding to plot the results the following corrections had to be made :—

I. For changes in the temperature of the steel chamber during the time of an experiment. For temperatures below 26° C. a mercury-thermometer labelled A was used, situated as described on p. 245, and 1 millim. change in A was equivalent to 4·05 millim. in the bridge-wire reading. Thermometer II. was used for temperatures above 26° C., and 1 millim. change in II. was equivalent to 5·5 millim. of the bridge-wire. The resulting correction had to be added or subtracted from the bridge-wire value of $d\theta_1$, and it rarely exceeded 0·2 millim.

II. The correction to the " mean bridge centimetre." This correction was given with great exactness by the table resulting from the calibration of the bridge-wire previously referred to. It is applied as a factor to $\dfrac{d\theta_1}{dt}$.

III. The normal rate of stirring during experiments 1 to 8 was 9·10 revolutions per second, for the remaining experiments 5·00. A considerable number of experiments gave $tr^2 = 800,000$ approximately, where r is the rate per second, and t the time of rising 1 centim. of the bridge-wire. The correction to the normal rate therefore is $\dfrac{r^2 - r_1{}^2}{800,000}$, where r is the normal and r_1 the observed rate.

As the stirring rarely departed much from the normal this correction was always small, it in no case exceeded ·00006, and rarely exceeded ·00002 ; and as the values of tr^2 given by the stirring experiments did not differ by more than 1 part in 50, the correction is sufficiently close.

IV. The correction consequent on changes in the temperature of the bridge-wire. This is given with sufficient accuracy by the expression

$$\frac{d\theta_1}{dt} \times \{1 + ·00016(\theta_1 - 59)\},$$

θ_1 being measured by a Fahrenheit thermometer.

V. The correction for the temperature of the Clark cells. Assuming Lord Rayleigh's temperature-coefficient ·00077, we get $\dfrac{d\theta_1}{dt}\{1+\cdot00154(\theta-15)\}$; for the rate of rise varies as E^2.

VI. The subsequent operations are simplified if some arbitrary value is assigned to R′, and the consequent value of $\dfrac{d\theta_1}{dt}$ for that value of R′ ascertained. As the resistance of the coil increased from about 8·45 to 8·6 ohms as θ_1 increased from 15° to 50° C., it was found convenient to take 8·5 ohms as the arbitrary resistance. The correction is $\dfrac{d\theta_1}{dt}\times\dfrac{R_1}{8\cdot5}$.

The true value of R_1, however, is not that obtained by merely taking the resistance in the usual manner with a small current. I have in paper J, pp. 404–7, published a full account of a method of finding the increase in R′ due to an increase in the potential-difference of 1, 2, &c. Clark cells.

The increase, with this coil, was carefully determined, with the following results*:—

<div align="center">

TABLE II.

</div>

Potential-difference (in terms of a Clark cell).	δR (in true ohms).
1	·00087
2	·00227
3	·00433
4	·00723

The resistance of the coil was carefully determined for each group of experiments, at the mean temperature of the group, and also occasionally at a fixed temperature, in order to trace any change. During the first series a slight increase was observable (apart from the natural increase due to its

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* I have to thank Mr. F. H. Neville, Fellow of Sidney College, Cambridge, for his kindness in assisting me in this somewhat troublesome determination.

temperature-coefficient).   This was possibly a consequence of an oversight on my part, as I had forgotten to previously anneal the wire.   After it had been raised to the highest temperature attained during these experiments, the further change became negligible.

The absolute value of R_1 was obtained by means of the dial-box described in paper J, pp. 408–10.   The errors of the separate coils and of the coils in the bridge-arms had been ascertained by a direct comparison, which had been made by kind permission of Mr. Glazebrook, with the B.A. standards in the autumn of 1892.

VII. The value of $\frac{d\theta_1}{dt}$ at several points of the range (which was usually about 15 centim. of the bridge-wire, $i.$ $e.$ about $1°\cdot4$ C.) having been ascertained, [the results were plotted (with θ_1 as abscissa) in order to ascertain the regularity &c. of the observations, which was generally found to be satisfactory.   The values of $\frac{d\theta_1}{dt}$ at the readings 60 and 70 of the bridge-wire were then obtained and divided by n^2.   Three experiments having been performed at each temperature with three different values of n, the intersection of the three lines thus found gave θ_N, the ordinate for that abscissa giving the rate of rise due to the electrical supply alone.   A plan of these intersections is given on Plate VIII.

A glance at Plate VIII. will show that the null point in each case is not accurately defined.

The intersection of the 3- and 4-cell lines is always to the left of the other intersections.

I have shown (p. 255) that the error introduced·by this uncertainty is in any case small, more especially so as the results are deduced from differences in the ordinates.   The most probable position appears to me to be about midway between the points of intersection of the 2-cell line with the other lines ; but as it is improbable that the position of the point would vary capriciously as the temperature rose, I have selected points on the smoothed curve (called the " null-point curve ") which passes most nearly through the various intersections.

It will be noticed that the null-point curve approaches more nearly to the outside temperature as the value of θ_1 increases[*]. This indicates either (1) that the work done by the stirrer had diminished (this would probably be due to diminished viscosity or surface friction) ; or (2) that the loss by convection, &c., had increased. The former is I think the more probable explanation.

When the depth of the liquid is increased, the stirrer has more work to do and the supply of heat is greater while the loss remains unchanged. The null point therefore is at a higher elevation when the mass is greater, and Pl. VIII. shows that $\theta_N - \theta_0$ is nearly proportional to the depth, *i. e.* to the mass, of the contained liquid.

In order to illustrate the method of applying the various corrections, and also to indicate their comparative magnitude, I give in full (Table III.) the working out of experiment 26, of which the details were given in Table II. The numbers in Roman numerals indicate the sections on the preceding pages, in which the particulars of the correction are given. The greatest correction is that given by the calibration of the bridge-wire, and this can be applied with great certainty.

I do not consider it necessary to crowd this paper with similar details regarding the remaining experiments. Table IV. (pp. 262, 263) is a summary in which I have given the numbers necessary for the remaining calculations.

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[*] This curvature is more marked than would at first sight appear from an inspection of Plate VIII. Owing to the difference in the temperature-coefficient of the two differential thermometers, the true bridge null point is given by the expression $59\cdot84 + \cdot0003\,\theta_1$, where θ_1 is the temperature of the thermometers. The position of the bridge null point is shown by the vertical lines on the left.

TABLE III.

Showing the Corrections necesssary for the Reduction of Experiment 26. July 28.

	Range on bridge-wire 55·5–62·5 cm.	Range on bridge-wire. 62·5–69·0 cm.	Range on bridge-wire. 69·0–76·0 cm.	Ref. No.
Observed time over range (t)	252·5 secs.	237·1 secs.	261·6 secs.	
Change in external temp. (therm. A) during the experiment	+·05 mm.	+·015 mm.	−·010 mm.	
Change in range consequent on change in therm. A (1 mm. A= 4·05 mm. of bridge-wire)	+·020 cm.	+·006 cm.	−·004 cm.	I.
Uncorrected rise per 1 sec., i. e. $\frac{d\theta_1}{dt}$	·027802	·027440	·026743	
Mean value of 1 cm. of the bridge-wire over this range in terms of mean bridge-wire cm.	1·00245	·99281	·99494	
Correction of $\frac{d\theta_1}{dt}$ to mean bridge-wire cm.	+·000068	−·000197	−·000135	II.
,, ,, for temp. of Clark cells.	+·000010	+·000010	+·000009	V.
,, ,, for temp. of bridge-wire.	+·000037	+·000037	+·000036	IV.
,, ,, for stirring to rate 5·00.	+·000012	+·000011	+·000011	III.
$\frac{d\theta_1}{dt}$ after corrections	·027929	·027301	·026664	
Mean point of range (bridge-wire)...	59·00	65·75	72·50	
	A.	B.	C.	

Since the points thus found should lie on a straight line, the best result is arrived at by treating the results so as to obtain 2 points only. If we mean A and B, and again B and C, we attach undue value to B. The true mean is obtained by $\frac{2A+B}{3}$ and $\frac{B+2C}{3}$.

We thus get :—

Bridge-wire readings .... 61·25 cm.   70·25 cm.

Mean value of $\frac{d\theta_1}{dt}$ .... ·027720   ·026877

We can thus obtain (either arithmetically or by plotting) the values of $\frac{d\theta_1}{dt}$ at B.W. readings .... 60·00 cm.   70·00 cm.

which are .... ·027837   ·026900

Now the value of R_1 near the "null point" (about 62·5 b.w.) was found to be 8·46093 true ohms. To this add ·00720 for the rise in resistance due to a potential-difference of 4 Clark cells (Table II.) ; hence $R_1=8·4681$, hence correction to R=8·5000 is − ·000104 .... VI.

Hence $\frac{d\theta_1}{dt}$ at B.W. readings .... 60·00 cm.   70·00 cm.

when R is 8·5000, = ... ·027733   ·026796

and $\frac{d\theta_1}{dt} \times \frac{1}{4^2}$ = ... ·001733   ·001675

TABLE IV.

Expt.	Date.	Mass of Aniline (in vacuo).	Number of cells. (n.)	Normal rate of stirring.	Mean external temp. C.	Temp. of "null point" above exterior. ($\theta_N - \theta_r$.)	External temp. + diff. to "null point." (θ_N.)	$\frac{d\theta_1}{dt} \times 10^6$ after corrections I., II., III., IV., V., VI. at		$\frac{d\theta_1}{dt} \times \frac{1}{\bar{n}^2} \times 10^6$. (Plate VIII.) at		Reading on "null-point" curve.	Mean value $\frac{d\theta_N}{dt} \times 10^6$
								60 b.w.	70 b.w.	60 b.w.	70 b.w.		
1, 2, & 3. 4 & 5.	July 12 ,, 13	294·99 ,,	3 2	9·10 ,,	16·935 16·908	+·844 +·844	17·847 17·720	25672 11935	24296 11585	2853 2984	2700 2646	2747 2750	2748
6 & 7. 8.	July 16 ,, 16	358·20 ,,	3 2	9·10 ,,	25·072 25·069	+·862 +·862	25·934 25·931	22929 10656	21397 9485	2547 2664	2377 2371	2390 2393	2392
9.	,, 24	,,	2	5·00	16·375	+·184	16·559	9923	8617	2481	2154	2417	2417
10 & 11.	,, 17 ,, 24	,,	3	,,	16·380	+·184	16·564	21998	20763	2444	2296	2415	
12.	,, 18	,,	4	,,	16·379	+·184	16·563	38973	37732	2436	2358	2420	
13. 14. 15.	,, 18 ,, 18 ,, 18	,, ,, ,,	3 2 4	,, ,, ,,	25·236 25·351 25·274	+·184 +·184 +·184	25·420 25·535 25·458	21735 9787 38658	20476 8677 37348	2419 2447 2415	2275 2169 2334	2391 2395 2400	2395
16. 17. 18.	,, 20 ,, 20 ,, 20	,, ,, ,,	2 3 4	,, ,, ,,	32·979 32·977 32·968	+·183 +·183 +·183	33·162 33·160 33·151	9717 21496 38168	8448 20255 36858	2429 2394 2385	2112 2256 2303	2370 2370 2370	2370
19. 20. 21.	,, 21 ,, 21 ,, 21	,, ,, ,,	3 2 4	,, ,, ,,	40·349 40·368 40·379	+·183 +·183 +·183	40·532 40·551 40·562	21127 9602 37885	19849 8357 36475	2367 2401 2366	2222 2089 2280	2340 2348 2348	2345
22. 23. 24.	,, 22 ,, 22 ,, 22	,, ,, ,,	3 2 4	,, ,, ,,	50·225 50·214 50·211	+·182 +·182 +·182	50·407 50·396 50·391	20726 9452 37317	19496 8256 36057	2336 2363 2332	2198 2064 2253	2311 2312 2318	2314

Table IV. (continued).

Expt.	Date.	Mass of Aniline (in vacuo).	Number of cells (n.)	Normal rate of stirring.	Mean external temp. C.	Temp. of "null point" above exterior, $(\theta_N - \theta_1.)$	External temp. + diff. to "null point." (θ_N)	$\frac{d\theta_1}{dt} \times 10^6$ after corrections I., II., III., IV., V., VI. at 60 b.w.	$\frac{d\theta_1}{dt} \times 10^6$ at 70 b.w.	$\frac{d\theta_1}{dt} \times \frac{1}{n^2} \times 10^6$ (Plate VIII.) at 60 b.w.	$\frac{d\theta_1}{dt} \times \frac{1}{n^2} \times 10^6$ at 70 b.w.	Reading on "null-point" curve.	Mean value $\frac{d\theta_N}{dt} \times 10^6$
25.	July 28	569·53	3	5·00	16·448	+·241	16·689	15665	14834	1741	1648	1710	
26.	,, 28	,,	4	,,	16·459	+·241	16·700	27733	26796	1733	1675	1713	1711
27.	,, 28	,,	2	,,	16·484	+·241	16·725	7135	6196	1784	1549	1710	
28.	,, 28	,,	2	,,	25·216	+·237	25·453	7050	6230	1763	1558	1695	
29.	,, 29	,,	3	,,	25·219	+·237	25·456	15557	14627	1729	1325	1696	1696
30.	,, 29	,,	4	,,	25·217	+·237	25·454	27426	26460	1714	1654	1696	
31.	,, 29	,,	2	,,	32·985	+·236	33·221	6966	6142	1742	1536	1679	
32.	,, 31	,,	4	,,	32·999	+·236	33·235	27245	26183	1703	1643	1684	1680
33.	,, 31	,,	3	,,	32·999	+·236	33·235	15398	14412	1711	1601	1678	
34.	Aug. 1	,,	2	,,	40·313	+·232	40·545	6890	6067	1722	1517	1663	
35.	,, 1	,,	3	,,	40·313	+·232	40·545	15222	14296	1691	1588	1662	1664
36.	,, 1	,,	4	,,	40·313	+·232	40·545	26983	25898	1686	1619	1666	
37.	,, 2	,,	3	,,	50·302	+·228	50·530	15015	14053	1668	1561	1639	
38.	,, 2	,,	2	,,	50·274	+·228	50·502	6804	5959	1701	1489	1640	1640
39.	,, 2	,,	4	,,	50·312	+·228	50·540	26535	25600	1658	1600	1641	
40.	,, 3	,,	3	,,	16·522	+·240	16·762	15701	14806	1745	1645	1712	
41.	,, 3	,,	4	,,	16·549	+·240	16·789	27708	26766	1732	1673	1712	1712
42.	,, 3	,,	2	,,	16·532	+·240	16·772	7120	6200	1780	1555	1713	

The value of $\frac{d\theta_N}{dt}$ having been ascertained as above indicated, the capacity for heat of calorimeter and contents is easily deduced.

The operations are shown in Table V.

Col. I. gives the temperature of the null point in degrees C. of the air-thermometer (from Table IV.).

„ II. the rate of rise at the null point as deduced from the group of experiments at that temperature (from Table IV.).

„ III. the value of the mean centimetre of the bridge-wire at the temperature of the null point, denoted by C_b.

„ IV. gives the rate of rise when $R = 1$ true ohm and the potential-difference 1 volt ($e = 1\cdot4342$, $e^2 = 2\cdot0570$), expressed in degrees C. of the air-thermometer. The value of R′ used in this reduction is $8\cdot4966$ instead of the $8\cdot5$ ohms which was selected as a convenient arbitrary value of R′ in the reductions in Table IV. The two wires connecting the roof of the calorimeter with the steel lid had a total resistance of $\cdot0068$ ohm. We may assume that half the heat generated in these wires passed into the steel lid, and half into the calorimeter. We may therefore consider their resistance as $\cdot0034$ ohm $= r$. Now the points which were kept at a constant P.D. were at the lid of the calorimeter and between these wires and the coil. The equation is J.H. $= \frac{E^2}{R_1}\left(1 + \frac{r}{R_1}\right)t$; hence the effective resistance $R_1 - r = 8\cdot5 - \cdot0034$.

„ V. gives the reciprocals of the numbers in Col. IV.

„ VI. The value of J is assumed as $4\cdot198$ *. The numbers in this column give the capacity for heat of calorimeter and contents expressed in terms of "a thermal unit at 15° C." The results when plotted as ordinates, with θ_1 as abscissa, are shown in Plate IX (A).

* Proc. Roy. Soc. vol. lv. p. 24 (1894).

TABLE V.

Experiments.	I. θ_N	II. $\dfrac{d\theta_N}{dt} \times 10^6$	III. C_b	IV. $\dfrac{d\theta_N}{dt} \times \dfrac{R' \times C_b}{e^2} \times 10^7$	V. T	VI. $\dfrac{T}{J}$
1 to 5 ...	17·784	2748	·09040	10265	974·2	232·05
6 to 8 ...	25·933	2392	·09062	8952	1117·0	266·08
9 „ 12 ...	16·562	2417	·09037	9020	1108·6	264·10
13 „ 15 ...	25·471	2395	·09061	8964	1115·6	265·77
16 „ 18 ...	33·158	2370	·09081	8890	1124·9	267·97
19 „ 21 ...	40·548	2345	·09102	8816	1134·3	270·22
22 „ 24 ...	50·398	2314	·09129	8722	1146·5	273·12
25 to 27 ...	16·705	1711	·09037	6387	1565·7	372·99
28 „ 30 ...	25·454	1696	·09061	6348	1575·3	375·30
31 „ 33 ...	33·230	1680	·09082	6300	1587·1	278·10
34 „ 36 ...	40·545	1664	·09102	6251	1599·8	381·11
37 „ 39 ...	50·524	1640	·09129	6185	1616·8	385·14
40 „ 42 ...	16·780	1712	·09037	6390	1564·9	372·80

Remarks on Tables IV. and V.

Experiments 1 to 3 (3 cells) and 4 to 5 (2 cells) were per-formed with a view to testing the working of the various portions of the apparatus, and in order to ascertain the minimum quantity of liquid that could be used with safety. The results were not satisfactory : the values of $\dfrac{d\theta_1}{dt}$ at the same temperature, when deduced from the different experi-ments in which the conditions were similar, differed con-siderably, occasionally by as much as 1 per cent. The cause was evident : the depth of liquid was too small to give satis-factory results with this form of stirrer. Considerable difficulty was experienced in maintaining the potential balance, and it is possible that at times portions of the coils were uncovered. I therefore decided on increasing the depth of the liquid.

Experiments 6 to 8.—The mass of aniline was now increased by about 60 grms., and the observations became more satis-factory. These experiments were performed with a stirring rate of 9·10 per second. I had not yet decided upon the best method of ascertaining the null point, and unfortunately did not

perform any 4-cell experiments. When deducing the results, I assumed that the intersection of the 2- and 3-cell lines stood in the same relative position to the intersection of the 2- and 4-cell lines as was found in those cases where a 4-cell experiment had been performed.

The null point is of course (see Plate VIII.) in a very different position from that found when the stirring-rate was 5·00 ; and the close agreement between the results given by these three experiments at the high rate, and experiments 13 to 15 at the same temperature but with a slow rate, is a satisfactory proof of the validity of this method of finding θ_N.

Experiments 8 *to* 39 call for no special comment. They were all conducted with a stirring-rate of (approximately) 5·00; Nos. 9 to 24 with the same mass of aniline as 6 to 8. From No. 25 upwards the mass of aniline was increased to 569·53 grms.

On plotting the values of the capacity for heat of this mass of aniline and the calorimeter, the spot obtained from experiments 25 to 27 appeared in a somewhat abnormal position ; consequently, although I had considered the experimental work as completed, I decided to repeat these experiments, and Nos. 40, 41, and 42 give the result. They agree so closely with the corresponding group as to confirm the position originally assigned to the curve at that temperature (about 16°·7 C.).

The curves on Plate IX. (A) give the capacity for heat of the different masses of aniline together with the calorimeter, at different temperatures. Both of the curves resulting from the larger masses show a marked change between 15° and 25° C. It is worthy of notice that the specific heat as deduced from these curves shows little or no signs of any similar change, but that it appears very markedly in the water-equivalent curve, and in such a manner as to indicate that it was approaching a minimum. Had I not been able to determine the specific heats by methods independent of the behaviour of the calorimeter, I should have concluded that the temperature coefficient of the specific heat of aniline altered considerably at the lower temperatures.

This indicates the necessity for extreme caution in similar investigations.

Table VI. gives a summary of the results, obtained from the values given by the curves in Plate IX. (A).

TABLE VI.

Col.   I. gives the capacity for heat of calorimeter $+358\cdot20$ grms. (*in vacuo*) of aniline.

„   II. gives the capacity for heat of calorimeter $+569\cdot53$ grms. (*in vacuo*) of aniline.

„   III. is the difference between the numbers in Cols. I. & II., and is therefore the capacity for heat of $211\cdot33$ grms. (*in vacuo*) of aniline.

„   IV. the specific heat of aniline at the respective temperatures.

„   V. the water-equivalent of the calorimeter.

Temp. θ_1.	I. $S_1 M_1 + w_1$. $M_1 = 358\cdot20$.	II. $S_1 M_2 + w_1$. $M_2 = 569\cdot53$.	III. $S_1(M_1 - M_2)$, $M_1 - M_2$ $= 211\cdot33$.	IV. S_1.	V. w_1.
15	263·82	372·38	108·56	·5137	79·82
18	264·36	373·13	108·77	·5147	80·02
20	264·74	373·69	108·95	·5155	80·11
25	265·80	375·14	109·34	·5175	80·44
30	267·10	376·88	109·86	·5198	80·90
35	268·50	378·82	110·32	·5221	81·49
40	270·02	380·86	110·84	·5244	82·19
45	271·58	382·90	111·32	·5268	82·88
50	273·00	384·88	111·88	·5294	83·39
52	273·55	385·63	112·08	·5304	83·60

Experiments 1 to 5 give $\dfrac{T}{J} = 232\cdot05$ (Table V. *supra*) when $M_3 = 294\cdot99$, and $\theta_1 = 17^\circ\cdot78$ C.   From the above table we get $w_1 = 80\cdot01$ when $\theta_1 = 17^\circ\cdot78$.

Hence
$$S_1 M_3 + w_1 = 232\cdot05$$
$$w_1 = 80\cdot01$$

$$\therefore S_1 M_3 = 152\cdot04 \quad \text{hence } S_1 = \cdot5154 \ (cf. \text{ with } \cdot5146 \ supra).$$

Again, the early experiments with water (p. 245) gave :—

when	$\theta_1 = 17\cdot2$	$w_1 = 80\cdot1$
and	$\theta_1 = 17\cdot2$	$w_1 = 79\cdot8$

hence when $\theta_1 = 17\cdot2$ $\qquad w_1 = 79\cdot95$ (*cf.* with $79\cdot99$ *supra*).

Although I have referred to experiments 1 to 5 as unsatisfactory in themselves, the value obtained from them is (as above pointed out) in fair agreement with that deduced from the remaining experiments.

The same remark applies to the absolutely independent determinations of the water-equivalent when conducted with water itself, and they afford strong corroborative testimony as to the accuracy of the conclusions.

The following simple formula gives the specific-heat curve with sufficient accuracy:—

$$S_1 = 0 \cdot 5156 + (\theta_1 - 20) \times \cdot 0004 + (\theta_1 - 20)^2 \times \cdot 000002. \quad \text{(A)}$$

The following table gives the experimental results and those obtained from the above formula.

TABLE VII.

θ_1.	S_1 (Experimental Results).	S_1 (from formula A).
15°	·5137	·5137
20	·5155	·5156
25	·5175	·5176
30	·5198	·5198
35	·5221	·5221
40	·5244	·5244
45	·5268	·5269
50	·5294	·5294
52	·5304	·5305

I have made a careful search for records of previous determinations, but I have been unable to find any in addition to those given in Landolt and Börnstein's tables—which are as follows :—

Temp.	S_1.	Observer.
8° to 82°	·5120	Schiff.
12° to 138°	·5231	,,
12° to 150°	·464	Petit.

NOTE (*Sept.* 6, 1894).—Having unfortunately strained one of the leads when the tank was at a high temperature, the insulation between the coil and the steel chamber commenced to fall off on Sept. 1st. I have therefore most reluctantly been compelled to take the whole apparatus to pieces, in order to replace the ebonite insulators. On Sept. 3rd I withdrew

the whole of the aniline, and on examining it I found that in colour it had darkened considerably. The experiments described in this paper were completed on Aug. 3rd, and on Aug. 16th the apparatus was again set going, and was kept continually at work until Sept. 3rd. During this time electrical currents were continually passing through the coil and the stirrer must have made some millions of revolutions. The nature of the experiments that I was then engaged upon compelled close attention to the capacity for heat of the aniline, and I am convinced that even a small change in the specific heat could not have escaped my notice. It would appear, therefore, that the change indicated by this darkening was not of a nature to sensibly affect the specific heat.

Note, *Dec.* 7, 1894.—In a communication made to the Royal Society on November 22, Professor Schuster pointed out an error in my determination of the value of J, viz. that I had not made a necessary correction for the specific heat of the air displaced by the water; for the method I had adopted gave the difference in the rate of rise when a certain space was filled first with air and then with water. This correction raises my value of J by about 1 in 4000.

Now the specific gravity of aniline (1·02) but slightly exceeds that of water; and as the method described in the preceding pages also depends upon the measurement of differences in the rate of rise, the correction would eliminate and would, therefore, not affect the values of the specific heat of aniline as given in Table VII. Again, if there are any errors in the values of the electrical standards upon which my determination of J depends, they also would eliminate during the calculations by which the specific heat of aniline was ascertained. There can, therefore, be no doubt that the value assumed for J ($4·198 \times 10^7$) is the correct one to be used for the reduction of the observations on aniline.

Discussion.

Dr. Armstrong thought the author had made a particularly happy selection in aniline, for it could now be obtained in almost any quantity absolutely pure. When pure it did not discolour on exposure, and would probably be very satisfactory

as a standard liquid. He doubted whether any hydrocarbon could be better.

Prof. AYRTON congratulated the author on the extreme accuracy obtained. Recently he had arranged an experiment to determine the mechanical equivalent of heat by the electrical method, which gave very accurate results, without any corrections whatever being necessary.

Prof. S. P. THOMPSON thought the whole phraseology of specific heat required revising.

Prof. PERRY agreed with Mr. Griffiths that a name for "capacity for heat per unit volume" was greatly needed, and Mr. Lucas suggested "heat density," but this was not satisfactory.

Dr. SUMPNER said most text-books on physics attributed the advantage of the mercury thermometer to the low specific heat of mercury, whereas the capacity for heat per unit volume was the important factor.

Mr. WATSON enquired to what temperature the alloy which the author had used to connect glass to metal had been tested?

The PRESIDENT said the paper was of great importance, because it dealt with the application of electrical methods to thermometry. The mercury thermometer had been quite superseded for work such as had just been described.

Mr. GRIFFITHS, in reply to Mr. Watson, said the alloy had been used successfully between 10° and 62° C. It gave way at 71° C. He was glad to learn from Dr. Armstrong that aniline could now be got pure. Prof. Ramsay had written to say that he did not think that the slight impurities in ordinary aniline would have much effect on its specific heats.

Mr. BLAKESLEY asked if aniline could be taken as pure if it did not change colour on exposure.

Dr. ARMSTRONG, in reply, said, Yes ; if the boiling-point was also constant.

XXV. *The Influence of the Relative Volumes of Liquid and Vapour on the Vapour-Pressure of a Liquid at Constant Temperature. By* SYDNEY YOUNG, *D.Sc., F.R.S., University College, Bristol**.

THE question whether the vapour-pressure of a liquid at a given temperature depends on the relative volumes of liquid and vapour has been frequently discussed, and has been the subject of many experimental investigations. Within recent years extended researches on the relations between the temperatures, pressures, and volumes of several liquids have been carried out by Prof. Battelli, and he arrives at the conclusion that when, in a tube containing a perfectly pure liquid and its vapour, the volume is diminished and the vapour caused to condense, the vapour-pressure rises : in other words, that the smaller the relative volume of vapour the higher is the vapour-pressure.

These results are entirely opposed to those obtained by Dr. Ramsay and myself and, while referring to this question in a letter to the ' Philosophical Magazine ' last February, we ventured to characterize the conclusions of Prof. Battelli as incorrect, and to suggest that the error was due to the presence of small quantities of air or other impurity in the liquid examined and to the employment of insufficiently purified substances for heating-purposes.

In a letter published in the August number of the ' Philosophical Magazine ' Prof. Battelli adheres to the conclusion previously stated, and does not admit the existence of either of the sources of error suggested. " I would rather observe," he writes, " that, in order to observe such a phenomenon, an apparatus is necessary which enables us—as in my case—slowly to compress the vapour, and to maintain it for a time under constant pressure."

There can be no doubt, I think, that such an apparatus is required in order to decide *the question whether the phenomenon exists*, but it seems hardly necessary to point out that

* Read November 9, 1894.

the conditions described are fulfilled in the apparatus employed by Dr. Ramsay and myself.

During the present year I have been engaged in an investigation of the thermal properties of isopentane—a liquid, boiling at 28°, which can, by suitable treatment, be obtained in a pure state *.

A great number of determinations of the vapour-pressure of the liquid were made at various temperatures, and in many cases the volumes of liquid and vapour were read.   The results obtained prove conclusively that the vapour-pressures of isopentane are independent of the relative volumes of liquid and vapour; and it may be of interest to give all the observed vapour-pressures at two temperatures, with the corresponding volumes of vapour and liquid.

The same apparatus was employed as in my previous work on benzene and its derivatives, the esters, &c.: it is similar in principle though it differs somewhat in detail from that made use of by Ramsay and myself (Phil. Trans. 1887 A, p. 59).

The tube containing the isopentane was heated by the vapour from pure liquids (Trans. Chem. Soc. 1885, p. 640 ; 1889, p. 483) boiling under reduced pressure.

The pressures are corrected for (1) the difference in height of the columns of mercury in the tube containing the isopentane and in the air-gauge ; (2) the expansion of the heated column of mercury ; (3) the pressure of the column of isopentane ; (4) the deviation of air (in the air-gauge) from Boyle's law, as determined by Amagat.

Several series of determinations were made with different quantities of liquid in the tube.   As a rule four readings of pressure were taken in each series at each temperature.

It will be seen that although the relative volumes of vapour and liquid vary within very wide limits, there is no such corresponding variation in the vapour-pressures, the greatest difference from the mean value at 140° being slightly less than 0·1 per cent., and at 90° slightly greater—in no case outside the limit of experimental error.

It has happened occasionally in the course of this or previous investigations that a trace of air has entered the tube

* Full details of the method of purification adopted will be described later when the research is completed.

	Temperature $= 90°$.				
Series.	Volume of Vapour in c.c.'s.	Volume of Liquid in c.c.'s.	Ratio $\left\{\frac{\text{Volume of Vapour.}}{\text{Volume of Liquid.}}\right.$	Vapour-Pressure.	Difference from Mean.
II.	0·016	0·52	0·03	4280	−1
III.	0·025	0·20	0·12	4281	0
II.	0·105	0·52	0·20	4286	+5
I.	0·07	0·23	0·30	4281	0
,, ...	0·17	0·23	0·74	4280	−1
III.	0·17	0·19	0·89	4279	−2
I.	0·28	0·23	1·2	4281	0
III.	0·30	0·19	1·6	4277	−4
I.	0·38	0·23	1·7	4282	+1
III.	0·42	0·19	2·2	4281	0
	Temperature $= 140°$.				
II.	0·012	0·54	0·02	11636	−2
,, ...	0·06	0·54	0·11	11634	−4
III.	0·06	0·20	0·30	11643	+5
I.	0·08	0·24	0·33	11637	−1
IV.........	0·06	0·07	0·86	11638	0
I.	0·20	0·23	0·87	11636	−2
III.	0·20	0·18	1·11	11643	+5
V.	0·07	0·06	1·2	11634	−4
I.	0·32	0·21	1·5	11641	+3
III.	0·34	0·17	2·0	11649	+11
I.	0·43	0·20	2·2	11642	+4
III.	0·48	0·16	3·0	11647	+9
IV.........	0·23	0·05	4·6	11633	−5
V.	0·26	0·04	6·5	11635	−3
IV.........	0·39	0·03	13·0	11632	−6
V.	0·45	0·02	22·5	11636	−2
IV.........	0·56	0·02	28·0	11634	−4
V.	0·62	0·002	310·0	11632	−6

or has been left in the liquid, and in a very few cases (ethyl formate, propyl formate) a small quantity of permanent gas has been formed by partial decomposition of the liquid by prolonged heating at high temperatures. The presence of permanent gas (or of very volatile impurity) is clearly indicated by the increase of pressure required for the complete condensation of the vapour, and the following points have been noticed in such cases:—(1) The pressure does rise as the volume of vapour diminishes ; (2) the readings taken with diminishing volumes are higher than with increasing volumes. This may be readily explained; for when the whole of the substance is in the state of vapour the air is diffused uniformly through it, and when condensation is brought about by compression the air remains to a large extent in the vapour above the liquid and dissolves but slowly in the liquid formed consequently, as the volume of vapour diminishes the undissolved air becomes more and more compressed, and its pressure increases and becomes considerable when the vapour is nearly all condensed. Similarly, with an impurity more volatile than the pure substance, a partial "fractionation" of the liquid takes place, the less volatile pure substance condensing first, whilst the vapour becomes more and more contaminated with the more volatile impurity.

On the other hand, when the readings are taken with increasing volume, the air or volatile impurity is at first uniformly distributed through the liquid, and it escapes but slowly through the long column of liquid into the vapour above. Thus when a reading is taken with a relatively small volume of vapour, the air present has a much greater influence in the first case than the second, but when the volume of vapour is relatively large its influence is small in both cases. I have noticed repeatedly that the freer a liquid is from air or permanent gas, the more closely do the readings during compression and expansion agree.

The presence of a small quantity of impurity less volatile than the pure substance may escape detection so long as the volume of liquid, relatively to that of the vapour, is fairly large ; but its influence becomes very marked when the . quantity of liquid is relatively very small. Thus, on one occasion, the isopentane had come in contact for a moment

with an indiarubber tube and had, no doubt, dissolved a minute quantity of it. It was noticed in this case that the mercury was slightly tarnished (owing in all probability to the sulphur in the indiarubber), but the vapour-pressures, when there was plenty of liquid present, agreed among themselves and with previous determinations. When, however, the volume was increased until nearly the whole of the liquid was evaporated, the observed vapour-pressures were far too low, and diminished rapidly as the volume was increased.

In this case and in those in which the presence of any permanent gas was detected, the experimental results were rejected and the tube was refilled.

DISCUSSION.

The CHAIRMAN remarked that, although the *à priori* considerations against Prof. Battelli's view were very great, it was satisfactory to have had the experimental enquiry carried out with such care and accuracy by Dr. Young.

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XXVI. *The Significance of Wiener's Localization of the Photographic Action of Stationary Light-Waves.* By J. LARMOR, *F.R.S., Fellow of St. John's College, Cambridge*.*

THE experiments by which Wiener demonstrated † that, when stationary plane-polarized optical undulations are produced in a photographic film, by reflexion of a stream of incident plane-polarized light at a metallic or other backing, the photographic action occurs at the antinodes of Fresnel's vibration-vector and not at the nodes, have been employed by its author and others to decide between the various theories of light. If for purposes of precise description we utilize the terminology of the electric theory of light, which formally includes all the other theories by proper choice of the vibration-vector, we may say that the photographic action takes place at the antinodes of the electric vector which corresponds

* Read November 9, 1894.
† Wiedemann's *Annalen,* 1890.

to Fresnel's vibration, and not at the intermediate antinodes of the magnetic vector which corresponds to MacCullagh's and to Neumann's vibration.

The crucial experiment of Wiener relates to the case when the angle of incidence is half a right angle, so that the direct and reflected waves which interfere are at right angles to each other. If the vibration take place along the direction of intersection of the two wave-planes, it will present a series of nodes and antinodes ; but if in the perpendicular direction there will not be such alternations of intensity. The experiment showed that when the light is polarized in the plane of incidence, the photographic plate develops a series of bands ; but when it is polarized in the perpendicular plane these bands are absent.

The argument employed is that the photographic effect will be greatest at those places in the stationary wave-train where the vibration is most intense ; and the conclusion is drawn from it that the actual vibration is represented by Fresnel's vector and not by MacCullagh's ; in other words, that the vibrations of polarized light are at right angles to the plane of polarization. The force of this argument, as against MacCullagh's theory, would, however, be evaded if the vector of that theory were taken to represent something different from the linear displacement of the æther, or if vibrations were excited in the molecule by rotation instead of translation, or by stress, as Poincaré has pointed out *.

But as a matter of fact it seems difficult to assign any reason of the above simple kind, on either theory, in favour of the photographic disturbance occurring at the antinodes rather than at the nodes of the optical vibration. The remarkable suggestion thrown out by Lord Rayleigh some time previous to Wiener's experiments, and afterwards verified by Lippmann, that certain effects in colour photography produced by Fox Talbot and Becquerel were really due to this kind of localization of the photographic effect, is not in opposition to such a view ; for the consideration adduced was simply that a localization, periodic with the waves, would, if it happened to exist, produce effects like the observed ones. At any rate,

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* See a discussion on this subject in *Comptes Rendus*, cxii. 1891, in which MM. Cornu, Poincaré, and Potier took part.

PHYSICAL SOCIETY OF LONDON.

ABSTRACTS OF PHYSICAL PAPERS

FROM FOREIGN SOURCES.

MARCH 1895.

GENERAL PHYSICS.

201. *Deformation of Zinc at different Temperatures.* **E. Ziman-sky.** (Wied. Ann. 54. pp. 139–163, 1895.)—The author fixes zinc rods, 330 mm. long, 5 mm. thick, fitted at both ends with brass caps 12 mm. long, within a copper cylinder which is filled with oil; a ring-shaped Bunsen burner surrounds the bottom of the cylinder. The rods are stretched by means of weights hanging over pulleys, and twisted at the same time by G. Wiedemann's torsion device. Particular care is taken to avoid any bending or sudden shocks. With this object, the torsion load is applied by means of a gentle jet of water flowing into a graduated beaker suspended from pulleys. The gradually heated rod under-goes spontaneous torsion. The rods contain 99 per cent. of Zn, 0·1 of Pb, and traces of Fe, As, Cd, S. They are used hard-drawn from the cast metal, and re-cast. The maximum stretching-loads are 142 kilog. for the hard-drawn material, 5 kilog. for the cast rods; the torsional loads are 1 kilog. and 0·5 kilog. Both torsion and untwisting are observed. On an average, the rods show a permanent stretching of 7 mm. and total temporary stretching of 9·3 mm. The angle of torsion reaches 45° in a few instances only. Characteristic differences in the behaviour of the hard-drawn and cast rods, slowly or quickly cooled, are hardly perceptible. Zimansky differs from some of the previous workers, but he follows his own method. He is inclined to hold, with Heinemann, that zinc becomes crystalline even during its manufacture. He does not find the peculiarities which Kalischer had observed in rolled zinc near 150°. According to

Holley, the temperature to which the zinc has been exposed in casting is of importance. Zimansky had no accurate information regarding this point. _____ H. B.

202. *The Chilo-Argentine Earthquake of October 27, 1894.* **A. F. Noguès.** (C. R. 120. pp. 167–170, 1895.)—The earthquake affected the eastern part of the Argentine Republic and the northern part of Chili, and was characterised by great intensity, long duration of the shock, amplitude of the oscillations, and the absence of subterranean noise. The epicentre was at San Juan and La Rioja. The zone of maximum intensity was an ellipse whose major axis passed in an approximately N. to S. direction through La Rioja, San Juan, and Mendoza: this epicentral zone of maximum intensity comprises also Uspallata and Albardon. The zone of medium intensity extended from Illapel, Oralle, and Santiago in Chili over to San Luis and Cordoba on the east of the Andes. Buenos Ayres, the north of the Argentine, and the Pacific on the west were in the zone of minimum intensity. At Rioja, San Juan, and Mendoza the first and most violent concussion occurred at $4^h 10^m$ P.M., and was followed by slighter shocks at $4^h 20^m$, $5^h 25^m$, 10^h, and 1^h A.M ; the first shock lasted 55 seconds. At Santiago it was first felt at $4^h 7^m 30^s$, and lasted 100 seconds; the seismograph indicated 2·5 cm. At Buenos Ayres it was observed at $5^h 2^m$—local time in both cases. In both places the movement was like the rolling of a vessel. The main direction seems to have been that of the major axis, somewhat N.E. to S.W., but E. to W. and N. to S. were also observed. At San Juan and Santiago it was rather a trepidation, without any particular direction. Lives were lost, and a great deal of damage done to buildings, water-courses, &c. According to one (untrustworthy) calculation made at Santiago, the wave travelled east with a velocity of 3000 m. per second. The Andes did not act as a barrier. The barometer had fallen before the shocks, and continued to do so while they lasted ; then followed, with rising barometer, a cold breeze from the south and stormy weather. The author is trying to connect the perturbations with the faults in the Andes chain, in accordance with Fouqué's views ; but the data are very meagre. In the zone of greatest disturbance the alluvium rests on ancient limestones and sandstones with trachy-andesites. H. B.

203. *Determination of Capillary Constants ; Remarks on the Work of Quincke.* **T. Lohnstein.** (Wied. Ann. 53. pp. 1062–1073, 1894).—The author deals chiefly with Quincke's recent contribution to Wied. Ann. (52. 1894) on the surface-tension of water and mercury, and, supporting himself by Sieg's and his own work, subjects some of the conclusions to very frank criticism. He also explains, mathematically, how to determine approximately the value of the surface-tension from Quincke's observations on air-bubbles, and shows that it does not disagree with the values obtained by other methods. _____ A. Gs.

204. *Integration of Equations of Diffusion with variable Diffusion Coefficients.* **L. Boltzmann.** (Sitz.-ber. math.-phys. Classe der k. bayer. Akad. der Wiss. 24. 3, 1894.)—This is a discussion of the questions raised in O. Wiener's paper (Wied. Ann. 49. 1893). Two liquids, whose indices of refraction are respectively n_2 for the upper and n_1 for the lower, are initially separated by the horizontal plane $x = 0$ and diffusion proceeds. If n be the index of refraction of the mixture at height x measured downwards at time t, n is a function of x and t.

In Wiener's experiments light from a metal plate is made to pass through a lens C, and so to fall perpendicularly on a vertical screen, the image of a line AB on the plate being a line $S_0 S S_1$ o the screen inclined 45° to the horizon. When a narrow glass vessel containing the diffusing liquids is interposed between th lens and the screen, owing to the variation with x of the index of refraction in the vessel, the image of the line on the screen appears as a curve $S_0 P S_1$, called the Curve of Diffusion. To every point P in this curve corresponds a particular value of x, namely, the height at which the light which falls on the screen at P entered the glass vessel. Wiener shows (see p. 125 of his paper) how by observation of the curve at any instant we can find n, $\dfrac{dn}{dx}$, and $\dfrac{d^2n}{dx^2}$ for any value of x at that instant. Also using the equation $\dfrac{dn}{dt} = k\dfrac{d^2n}{dx^2}$ with k constant, he determines k, in terms namely of t, the time which has elapsed since the beginning of the diffusion, if that be known, or if not then in terms of t_1 and t_2 two different times of observation.

Wiener subsequently (p. 143) obtains the more general equation

$$\frac{dn}{dt} = K\frac{d^2n}{dx^2} + \kappa\left(\frac{dn}{dx}\right)^2,$$

in which κ is constant and $K = K_0 + \kappa\overline{n - n_0}$, but does not give the integral of this equation.

Boltzmann gives the solution of the equation

$$\frac{dn}{dt} = \frac{d}{dx}\left(k\frac{dn}{dx}\right)$$

with variable k. This solution, originally due to Boltzmann, is given by Hausmaninger (Sitzungsberichte der Wiener Akad. 86. 1882). It is here re-stated in the form

$$n = n_2 - \frac{(n_2 - n_1)\displaystyle\int_{\frac{x}{\sqrt{t}}}^{\infty}\frac{d\lambda}{n}\,\epsilon^{-\int_0^y \frac{\lambda d\lambda}{2k}}}{\displaystyle\int_{-\infty}^{\infty}\frac{d\lambda}{-}\,\epsilon^{-\int_0^\lambda \frac{\lambda d\lambda}{2k}}} \quad . \quad . \quad . \quad . \quad (2)$$

Boltzmann applies this solution, using Wiener's formulæ, to the determination of k in the form

$$k = \frac{\eta^2}{2tz} \int_y^\infty z(y - y_0) dy,$$

in which $y - y_0$ is the vertical height of S_0 above S, and η the ratio between the distance of the screen from the lens C and the distance of the vessel from C.

Then, dealing with cases in which the equation

$$\frac{dn}{dt} = \frac{d}{dx}\left(k\frac{dn}{dx}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

cannot be assumed to be accurate, he starts from

$$\frac{du}{dt} = \frac{d}{dx}\left(x\frac{du}{dx}\right),$$

in which u is the *concentration* of either liquid at x. And assuming

$$\frac{du}{dn} = h \text{ (constant), and } k = hx,$$

he obtains for (2) the new form

$$n = n_2 - \frac{(n_2 - n_1)\int_{\frac{x}{\sqrt{t}}}^\infty \frac{d\lambda}{k} \, \epsilon^{-\int_0^\lambda \frac{h\lambda d\lambda}{2k}}}{\int_{-\infty}^\infty \frac{d\lambda}{k} \, \epsilon^{-\int_0^\lambda \frac{h\lambda d\lambda}{2k}}} ; \quad . \quad . \quad . \quad (2\,a)$$

and this leads to

$$k = \frac{\eta^2}{2\,tz} \int_y^\infty hz\,(y - y_0)\,dy.$$

If h be variable, a further correction is required. It is shown that the equation will then be

$$\frac{z^2}{a\delta}\frac{1}{h}\frac{dk}{dn} + \frac{1}{\eta}\frac{dz}{dy}\frac{k}{h} + \frac{\eta}{2\,t}(y - y_0)z = 0,$$

from which have to be found the two unknown quantities $\frac{k}{h}$ and $\frac{1}{h}\frac{dk}{dn}$ by observations of the curve at two points of time t_1 and t_2, if the time t since the beginning of diffusion be known, or at three points if it be unknown. In the last equation $a\delta$ is a constant defined in Wiener's paper.   S. H. B.

205. *Proof of Maxwell's Law of the Distribution of Velocities among Molecules of a Gas.* (Wied. Ann. 53. pp. 955–958, 1894.) *The Ratio of the Specific Heats of Gases at Constant Volume and Constant Temperature.* **L. Boltzmann.** (Sitz.-ber. k. bayer. Akad. Wiss. Bd. 24. Heft 3, 1894.)—Boltzmann here points out an error into which Kirchhoff has fallen, treating the chances for two mole-

cules having their momenta and coordinates within given limits as independent, and therefore the chance for the two as the product of the separate chances F and f, immediately *after* an encounter. That may be true before, but cannot generally be true after, encounter. The point is important as bearing on the controversy which has been, or is now, going on as to the validity of Boltzmann's proof of the uniqueness of Maxwell's law. Boltzmann proved that until Maxwell's law is established a certain function H necessarily diminishes, and is then only constant when the law prevails. To which it was objected that if the velocities be all reversed, the whole system must retrace its course with H increasing, but nevertheless (it was said) the theorem still applies to prove its diminution. The answer is that the proof of the theorem is founded on the assumption that the chances F and f are independent, and, as here shown, in the reverse motion they are not independent.

A point of greater physical interest, the ratio of the specific heats, is raised by reference to Pringsheim's experiments (Wied. Ann. 49. p. 347, 1893). Pringsheim finds that if lithium be heated in a porcelain tube, so circumstanced that no chemical action takes place, it gives, however high the temperature, no trace of the known lithium spectrum. But if the contents of the tube are such that chemical action does take place—according to Pringsheim between the lithium and the porcelain of the tube by the formation and reduction of lithium silicates—the characteristic spectrum at once appears. Boltzmann suggests another possible source of chemical action, viz. electrical oscillations of the surrounding ether. The inference we are asked to draw is that the lines in the spectrum of lithium or other substance are not connected with specific internal vibrations of the atoms or molecule of the substance, and teach us therefore nothing as to the number of degrees of freedom it may have. Therefore that the observed ratio between the specific heats of certain gases may, for all that the spectroscope tells us, be accounted for on the assumption of gas molecules being analogous to solid bodies with 3, 5, or 6 degrees of freedom—the ratios thence deduced being respectively 1·67, 1·4, and 1·33. The many modes of vibration shown by the spectrum correspond it would seem to degrees of freedom, not of the molecule, but of the ether. S. H. B.

206. *Improved Anemometer*. **J. Richard.** (C. R. 120. pp. 121–122, 1895.)—The transverse axis carrying the two anemometers measuring the horizontal and vertical components is brought into position by a central anemoscope, so that the two anemometers do not mask each other. The vanes of the anemometer for the horizontal component are portions of a cylinder, and give excellent results. In the electric connection, two of the seven wires hitherto used have been dispensed with. This anemometer is intended for the new Observatory erected by M. Decheverens on Jersey Island. E. E. F.

207. *Elasticity of Gelatine-Jellies.* **E. Fraas.** (Wied. Ann. 53. pp. 1074–1091, 1894.)—For purposes of experiment the jelly, made in general by dissolving gelatine in water, is cast into the form of rods of radius 7·85 mm., length 30 cm. The measurements made are based on the equation $\lambda = \epsilon PL/q$, where L is the length of rod under consideration, q the sectional area, P the stretching force, λ the elongation, and ϵ the elastic coefficient; it must be noticed *that ϵ is the reciprocal of Young's modulus.* The magnitude of ϵ is not a constant, but decreasing—with diminishing rapidity—with time, ultimately approaches a limiting value which is smaller as the quantity of gelatine per unit volume is greater. The addition of sodium chloride considerably increases the elastic coefficient; whilst glycerine, cane-sugar, and gum-arabic can, to a certain extent, replace the water without producing any change. The results obtained are explained by the use of Pfeffer's ideas on the rearrangement, with time, of the molecular constitution of colloidal substances.   A. G.

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208. *Freezing-Points of Mixtures of Liquids.* **R. Pictet** and **M. Altschul.** (Zschr. phys. Chem. 16. pp. 18–23, 1895.)—Mixtures of alcohol and water are made containing varying percentages of alcohol up to 80 %. The curve giving the F.P. in terms of the percentage of alcohol is a straight line up to 6·8 %. From 16·4 % to 30 % and from 39 % to 65 % it is also straight, but the lines in these cases when produced do not pass through the origin. In the intervals between the parts that are straight there is no obvious relation between the F.P. and the concentration. The authors conclude that between 16·4 and 30 and between 39 and 65 the lowering of the F.P. is proportional to the quantity of a definite hydrate of alcohol present.

In another series of experiments the mixtures are made by adding successive gramme-molecules of water to 1 gramme-molecule of alcohol. The curve giving the F.P. in terms of the number of molecules of water is approximately a hyperbola.   J. W. C.

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209. *Freezing-Points of some Organic Liquids.* **M. Altschul** and **B. von Schneider.** (Zschr. phys. Chem. 16. pp. 24–25, 1895.)—From observations on the F.P. of seventeen liquids—mostly benzene derivatives—the following results are deduced:—

    (1) Addition of CH_2 to an aromatic compound lowers the F.P.

    (2) Substitution of Cl for H in a CH_3 group raises the F.P.

    (3) In the case of homologous compounds of like structure the compounds with an odd number of carbon atoms in the molecule have lower F.P.'s than those with an even number.

    (4) Of isomers the compound with the more symmetrical formula has the higher F.P.   J. W. C.

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210. *Critical Temperature as a Test of Purity.* **M. Altschul** and **R. Pictet.** (Zschr. phys. Chem. 16. pp. 26–28, 1895.)—

The authors urge the claims of the critical temperature as a sensitive test of the purity of a liquid. If pure chloroform has a few drops of alcohol added to it, so as to change the boiling-point by $\frac{1}{10}°$, the critical temperature changes by 3·8°. Ethyl chloride has its critical temperature raised 6°, whilst the B.P. rises 1°. Commercial pentane has a critical temperature $1\frac{1}{2}°$ higher than pure pentane.   J. W. C.

211. *Freezing-Points of Concentrated Solutions.* **R. Abegg.** (Zschr. phys. Chem. 15. pp. 209–261, 1894.)—The laws for dilute solutions are extended to strong solutions, suitable corrections being made for the change in density which occurs for a large lowering of the freezing-point. The numbers are worked out both according to Arrhenius' and Raoult's method of reckoning concentration. Water, acetic acid, and benzene solutions give results which indicate that their ionising powers are in the order in which they are here written. All the solutions depart from the laws of dilute solutions. The "association" into polymeric molecules observed by Ramsay and Shields is used to explain the deviations in some cases.   S. S.

212. *Method of describing a Straight Line by means of Link-work.* **R. Bricard.** (C. R. 120. pp. 69–71, 1895.)—Any two fixed points O and O′ are joined by links OM, MP, PM′, M′O′. The two links OM, O′M′ have the same length R, and the two links MP, M′P the same length d. On MO and M′O′, produced if necessary, two points Q and Q′ are taken such that

$$MQ = M'Q' = d^2/R,$$

and are joined by a link QQ′ whose length equals OO′. d/R. On deforming the system, P moves in a straight line which is perpendicular to OO′ and passes through its middle point.

Convenient dimensions are

$$OO' = 2R, \quad d = R\sqrt{2}, \quad OQ = O'Q' = R, \quad QQ' = OO'\sqrt{2}.$$

The author concludes with a geometrical proposition which shows that the problem of describing a straight line by means of link-work has an infinite number of solutions.   A. Gs.

213. *Perforation of Metallic Plates by Projectiles.* **E. Vallier.** (C. R. 120. pp. 136–139, 1895.)—The subject is treated partly empirically, but to a large extent mathematically.   A. Gs.

214. *Utilisation of the Critical Point of Liquids for the Confirmation of their Purity.* **R. Pictet.** (C. R. 120. pp. 43–45, 1895.)—The author finds that, in the case of the liquids examined, a small quantity of impurity alters the critical point considerably, whilst the boiling-point is only affected slightly. To determine the critical point, a tube 45 to 50 mm. long, 5 mm. external and 3 mm. internal diameter, is partly filled with the liquid under

examination, then heated, and finally sealed at the upper end when it is about one-third full of liquid and two-thirds of vapour; it is then placed, beside a mercurial thermometer, at the centre of a thin sheet-iron cylinder provided with windows of mica. This is surrounded by another sheet-iron cylinder coated with asbestos.   The whole apparatus is heated at the bottom by means of gas flames, a certain amount of the hot products of combustion being allowed to pass between the cylinders. To ascertain the critical point the tube is slowly heated until the meniscus of the contained liquid disappears, then cooled until liquid forms.   It is again slowly heated and, after a few trials, a temperature is found such that an increase or decrease of one-tenth of a degree causes the meniscus to disappear or the liquid to form.

The results obtained are indicated in the Table given below:—

	Critical Point.	Change in Critical Point.	Change in Boiling-Point.
Pure chloroform	258·8°		
Chloroform mixed with a few drops of alcohol ...	255·0	−3·8°	−0·1 to 0·2°
Pure ethyl chloride	181·0		
Ethyl chloride mixed with a few drops of alcohol...	187·0	+6·0	+0·6
Pure pental	201·2		
Pental mixed with a few drops of aldehyde	199·5	−1·7	<0·1

A. Gs.

215. *Experimental Researches on the Critical Points of Liquids holding Solid Bodies in Solution.* **R. Pictet.** (C. R. 120. pp. 64–67, 1895.)—The paper contains an interesting and detailed account of the phenomena which occur when certain liquids in which solid bodies are dissolved are heated above their critical point. The author's description of the phenomena which occur when the liquid obtained by dissolving 0·5 gr. of borneol (an isomeric form of camphor) in 25 gr. of ether is heated, may be taken as a sample:—"At the moment when the temperature reaches the critical point of ether, at 189°, there appears, on the upper walls of the tube containing the solution, a condensation which descends towards the liquid below; the agitation of the liquid is extreme, much greater than with pure ether.   The temperature rises to 197° before the meniscus disappears and the critical point of the solution is reached.   All the liquid vanishes, but no solid deposit is left in the tube, which remains transparent.   The camphor is gaseous, or in a gaseous solution in the vapour of ether."   In addition, solutions of cineol, terpinol, guaiacol, and iodine in

ether, of borneol in ethyl chloride, of alizarin in alcohol, are examined ; finally there are some theoretical remarks.

The table given below summarises most of the results obtained :—

Mass in grams		Critical Point.	Variation of	
of body dissolved.	of the solvent.		Boiling-Point.	Critical Point.
gr. 0·5 of borneol ...	gr. 25 ether.........	○ 197	○ 0·2	○ 8·0
0·5 of cineol	25 ,, ...	193	0·2	4·0
0·5 of terpinol ...	25 ,, ...	197	0·2	8·0
45·0 of borneol...	55 ,, ...	296	not yet determined.	107·0
0·5 of phenol ...	25 ,, ...	201	0·1	12·0
0·5 of guaiacol ...	25 ,, ...	195	0·3	6·0
1·0 of iodine......	30 ,, ...	193	0·2	5·0
1·0 of borneol ...	50 ethyl chloride.	191	0·2	10·0

———— A. Gs.

216. *Dynamic Problems whose Differential Equations admit of an Infinitesimal Transformation.* **P. Staeckel.** (Ber. k. sächsischen Ges. der Wiss. zu Leipzig, M.P. Classe, 1893, Band iv.) *Note on the above by the same Author.* (C. R. 119. pp. 508–512, 1894.) *Further Note by the same Author.* (C. R. 119. pp. 723–725, 1894.)—In his " Untersuchungen über geodätische Curven deren Differentialgleichungen eine infinitesimale Transformation gestalten," Mathemat. Annalen, 1882, p. 357, Lie solved the following problem. To define the line-element of a surface which can be infinitesimally displaced (" in sich verschoben," *ibid.* p. 358) in such manner as that every geodesic curve passes into a neighbouring geodesic curve of the same surface. In the volume for 1892 of the Leipzig ' Berichte' is an article by O. Staude (p. 429) in which he considers the problem of the motion of a material particle constrained to move on a surface with a force function whose equation is $\phi(uv)=0$. Staude finds the condition that the differential equation of the motion shall admit of an infinitesimal transformation expressed by the symbol

$$\mathrm{T}f = \xi(u\,v)\frac{df}{du} + \eta(u\,v)\frac{df}{dv}$$

is that the surface belongs to the class of surfaces defined by Lie in the above work, and that the lines of constant potential on the surface are the lines of motion in the infinitesimal transformation. It is understood that the potential is not constant.

M. Staeckel, in the article in the Leipzig 'Berichte' mentioned above, generalises Lie's (or Staude's) problem. Instead of a single particle whose position is defined by the two variables $u\,v$, he considers a dynamical system of any number of material points whose positions are defined by n generalised coordinates, $p_1 \ldots p_n$. And these again may be represented by the motion of a single particle " in einer n-fach ausgedehnten Mannigfaltigkeit,"—an expression for which we have not as yet naturalised an English equivalent. Let the equation of energy be

$$\tfrac{1}{2} \sum_{\kappa\lambda} a_{\kappa\lambda}(p_1 \ldots p_n) \frac{dp_\kappa}{dt} \frac{dp_\lambda}{dt} = \Pi + h,$$

where Π is the potential and h an arbitrary constant. Herr Staeckel's propositions are, in his own words :—

I. "Jedes Niveaufeld, $d.\ h.$ jede $(n-1)$fache Mannigfaltigkeit constanter Kraftfunction, durch die infinitesimale Transformation in sich übergeführt wird";

II. "Die infinitesimale Transformation für die Mannigfaltigkeit conform (vergleich Lie, $loc.\ cit.$ p. 368) ist";

III. "Bei dieser conformen infinitesimalen Transformation auch die Schaar der ∞^{n-2} geodätischen Linien der n-fachen Mannigfaltigkeit invariant bleibt."

In the first note contributed to the 'Comptes Rendus' as above mentioned, M. Staeckel shows how to determine whether a given dynamical problem satisfies the above conditions or not. The condition that it may do so is :—It must be possible so to choose the variables $p_1 \ldots p_n$ as that

(1 a) Π shall be a function of $p_2\, p_3 \ldots p_n$ only ;

(2 a) The expression for the kinetic energy must admit of being reduced to

$$\tfrac{1}{2} e^{cp_1} \sum_\lambda b_{\kappa\lambda}(p_2 \ldots p_n) \frac{dp_\kappa}{dt} \frac{dp_\lambda}{dt},$$

where c is an arbitrary constant and $b_{\kappa\lambda}(p_2 \ldots p_n)$ is a function of $p_2\, p_3 \ldots p_n$ only.

In the second note (Comptes Rendus, pp. 723–725) he gives certain other results of his investigations on the same subject, as follows :—The differential equation of a dynamical system in two variables p_1 and p_2, of which the force-function is not constant, admits of only one infinitesimal transformation independent of the constant, h, of the equation of energy. Again, in order that the system of $n-1$ differential equations in $p_1\, p_2 \ldots p_n$ may admit a continuous group G of two parameters independent of the constant h, it is necessary and sufficient that the variables $p_1\, p_2 \ldots p_n$ may be so chosen that

(1) The force-function contains only $p_3\, p_4 \ldots p_n$;

(2) The expression for the kinetic energy can be reduced to one of the two forms following, viz. :—

(A.) $\quad \frac{1}{2} e^{\omega_1 p_1 + \omega_2 p_2} \sum_{\kappa\lambda} c_{\kappa\lambda}(p_3 \dots p_n) \frac{dp_\kappa}{dt} \frac{dp_\lambda}{dt},$

(B.) $\quad \frac{1}{2} p_2^{\gamma - \epsilon_{\kappa 1} - \epsilon_{\kappa 2} - \epsilon_{\lambda 1} - \epsilon_{\lambda 2}} c_{\kappa\lambda}(p_3 \dots p_n) \frac{dp_\kappa}{dt} \frac{dp_\lambda}{dt},$

where ω_1, ω_2, and γ are arbitrary constants, and

$$\epsilon_{\kappa\lambda} = 0 \text{ if } \kappa \neq \lambda, \quad \epsilon_{\kappa\lambda} = 1 \text{ if } \kappa = \lambda.$$

The infinitesimal transformations of the group G are

(A.) $\qquad \left| \begin{array}{cc} \dfrac{df}{dp_1}, & \dfrac{df}{dp_2} \end{array} \right|,$

(B.) $\qquad \left| \begin{array}{cc} \dfrac{df}{dp_1}, & p_1 \dfrac{df}{dp_1} + p_2 \dfrac{df}{dp_2} \end{array} \right|.$

<div align="right">S. H. B.</div>

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217. *Torsional Vibrations of a Wire.* **G. Berson** and **H. Bouasse.** (C. R. 119. pp. 48–50, 1894.)—An annealed platinum wire is thrown into torsional vibration by the action of a suitable impulse, and the positions of the wire at very short intervals of time determined photographically. It is found that, although the statical limit of elasticity is not approached, a permanent twist of the wire in the direction of its first motion is at once produced, and the vibrations are executed about this new position. The twist increases with the diameter of the wire and with the maximum acceleration during the action of the impulse. If when the vibrations have died down a second impulse equal to the first and in the same direction is given, the permanent twist is increased, while if the impulse is in the opposite direction the twist is diminished; but the change due to the second impulse is less than that produced by the first, and if more impulses are given similar results are obtained. It follows that such a wire loses a considerable amount of energy during its first oscillation. <div align="right">L. R. W.</div>

LIGHT.

218. *Absorption of Solar Radiation by Mist.* **A. Bartoli** and **E. Stracciati.** (N. Cim. 36. pp. 287–297, Dec. 1894.) *Transmission of Solar Radiation as affected by Atmospheric Dust.* **A. Bartoli.** (N. Cim. 36. pp. 183–189, Oct. 1894.)—The first of these papers gives the results of pyrheliometer observations made at Catania and on Etna under various atmospheric conditions. They show that a stratum of cirrhi may intercept as much as 30 per cent. of the solar radiation, which would be transmitted under the same conditions as to altitude of sun and hygrometric state. A light blue sky absorbs more than a deep azure sky, the ratio of the two absorptions ranging from 77 per cent. with the sun at 10°, to 96 per cent. with the sun at 50° above the horizon. A light mist uniformly distributed transmitted from 58 to 92 per cent. of the radiation from a deep azure sky.

The second paper deals with the influence of the impalpable dust due to the great eruption of Etna in July 1892. The light dust was uniformly distributed and fell very gently. There was no wind, and the sky was quite clear. The sun appeared slightly less brilliant than usual. The amount of heat transmitted ranged from 30 per cent. for an altitude of 12°, to 70 per cent. of the usual amount at 30°. In each case the quantity transmitted was expressed by an equation of the form $Qe^n = C$, where Q is the quantity of heat, e the thickness of air traversed, and n and C constants.   E. E. F.

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219. *Sensitiveness of Eye to Rhythmical Flashes.* **C. Henry.** (C. R. 120. pp. 147–149, 1895.)—The author distinguishes between rhythmic series, represented by the numbers 2^n, $2^n + 1$, $2^m(2^n + 1)(2^p + 1)$, and non-rhythmic series. A blackened drum, 1 m. radius, turned by clockwork, is perforated with 60 holes, which can be closed and opened. The drum revolves at three speeds: high, one revolution in from 7·5 to 13 sec.; medium, 70 to 82 sec.; low, 110 to 113 sec. The source of light is either an Auer burner, fed with oil-gas with a pressure regulator, or a very feeble oil-lamp with a screen of stout paper. Four series are used:—(1) successions of 60 apertures, 21 rhythmic, 44 non-rhythmical numbers (one hole common to both); (2) 30 apertures, one in two being closed, 15 rh., 16 non-rh.; (3) 20 apertures, two in three closed, 13 rh., 8 non-rh.; (4) 15 apertures, three in four closed, 10 rh., 6 non-rh. The variations of sensibility are estimated by determining the minimum perceptible before each experiment p_1, after each p_2, and by calculating $\dfrac{p_2 - p_1}{p_1} = P$, which expresses the variation of anæsthesia. The minimum is determined with the help of a diaphragm photometer which Henry had constructed by Radiguet, the source of light being a screen of phosphorescent

zinc-sulphide. As the variations in the brightness of this compound are known for different periods, the quantity of light which strikes the retina can be found from the aperture of the diaphragm corresponding to the minimum perceptible. The instrument is more exact for longer times and feebler lights; and as the quantity measured depends upon both time and aperture, there is no danger of the disturbing influence of foregone conclusions. Within the limits 1 to 80 seconds, the minimum perceptible decreases with the time during which the eye is kept in the dark; as the photometrical test requires several seconds, all the minima observed at different times have to be reduced to zero time. The following table gives P for the succession of 60 holes as determined by the right eye of Meunier, after a rest of 3 or 4 minutes. The rh. means rhythmic.

	High Speed.		Medium Speed.		Low Speed.	
	Rh.	Non-rh.	Rh.	Non-rh.	Rh.	Non-rh.
Strong light $\{$	..	..	..	..	1·07	−0·126
	..	..	..	..	1·08	−0·119
Feeble light.	0·559 $\left\{\begin{array}{c} -0·36 \\ -0·709 \end{array}\right\}$		0·17	−0·572	0·136	−0·593

P is therefore always negative for non-rhythmic series, with feeble light at all speeds and with strong light at small velocities. With strong light the persistence of the impression disturbs the phenomena. For the complex series of 60 apertures this rule is always confirmed; in other cases of less complex series different observers were uncertain, p_1 varying in the different experiments. Even after resting the eye in the dark for twenty-five minutes, it is found impossible to bring the retina back to the initial state. The experiments were made at the Dépôt des Phares, and tend to show that the range of a light-signal can be increased by producing flashes in a sufficiently complex, non-rhythmic succession. H. B.

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220. *Imitation of the Phenomena of Dispersion of Light. Theory of the Mutual Action of a Series of Pendulums and Air.* **E. Ketteler.** (Wied. Ann. 53. pp. 823–831, 1894.)—The paper opens with some remarks and references of a general character, but the greater part is devoted to a description and mathematical treatment of a model—of a character indicated by the second part of the title—designed to illustrate the connection which, in general, exists between anomalous dispersion and absorption. A. G.

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221. *Emission of Light.* (Wied. Ann. 53. pp. 832–840, 1894.) *A Note on the preceding.* (Wied. Ann. 54. pp. 178–180, 1895.) **G. Jaumann.**—In the first paper the author—reasoning on the supposition that, in many cases, the vibrations given out by a glowing body are subject to damping—gives some mathematical formulæ which he uses to explain the production of continuous, line, and band spectra. The second paper contains, amongst

other things, an acknowledgment that Lommel (Wied. Ann. 1877) had already explained the widening of bright lines of a spectrum by a similar supposition.   A. G.

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222. *Refractive Index of Sulphur and Phosphorus Solutions.* **V. Berghoff.** (Zschr. phys. Chem. 15. pp. 422–436, 1894.)— The refractive indices of solutions of sulphur and phosphorus in carbon bisulphide are determined by a modification of a method devised by Feussner. A prism with an angle of 60° is placed close to the objective of a telescope. Three metres away are placed two fixed straight scales, whose normals make angles of about 60° with the axis of the telescope. The angle of the prism and the minimum deviation are determined by observing the graduations on the scales which coincide with the cross wires in the telescope, when the light is reflected from the faces and refracted through the prism respectively. The paper gives details of the adjustments, the method of calculating the constants of the apparatus, and the formulæ required for finding the refractive index. The author uses his results to test the usual expressions connecting the refractive index and the density. He finds that $\frac{n-1}{d}$ is constant, both when the concentration of the solution changes and when the temperature changes. $\frac{n^2-1}{d}$ increases, and $\frac{n^2-1}{n^2+2} \cdot \frac{1}{d}$ decreases with increasing concentration. For solutions of sulphur the value of $\frac{n-1}{d}$ is ·4973, which is nearly the same as the value for solid sulphur. The results of the experiments do not confirm the formula of Pulfrich (Zschr. phys. Chem. 4. p. 561), giving the refractive index of the solution in terms of those of the constituents.   J. W. C.

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223. *Rotativity of some Active Amyl Compounds.* **P. Walden.** (Zschr. phys. Chem. 15. pp. 638–655, 1894.)—The author gives the rotativity of 26 derivatives prepared from the same sample of amyl alcohol, and examines the results with the object of testing Ph. Guye's law connecting the rotativity with the mass of the groups attached to the asymmetric carbon atom. The results lead to the conclusion that the mass of the groups is not the only factor, and that many other conditions, such as the nature of the radical or element, affect the rotativity.   S. S.

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224. *Huygens' Problem.* **B. Brunhes.** (J. Phys. 4. pp. 5– 20, Jan. 1895.)—The problem of Huygens consists in the determination of a distribution of luminous sources over a given closed surfree, which shall produce at points outside the surface the same illumination as a given system of luminous sources within the surface. Fresnel assumed a solution of it in his explanation of diffraction ; he considered every element of a

wave-surface as a source of light having the same motion as the element itself. Gouy has pointed out that Fresnel's hypothesis leads to two anomalies in the case of plane waves; the vibratory velocity at any point as calculated from the surface-elements is a quarter of a period behind the actual velocity, and the expression for it contains in the denominator the wave-length λ. He has shown how the hypothesis may be modified in order to furnish correct results for plane waves. The author, after referring to the work of Gouy, Kirchhoff, and Beltrami, treats the problem for longitudinal waves as the analogue of Green's problem in electricity; the surface-density of the electrical problem has in the acoustic case the dimensions of a velocity, electrical potential corresponding to velocity potential. Calling the velocity of the vibratory movement at any point of a plane wave $\sigma(t)$ at time t, and the velocity of propagation V, it is shown that the vibratory movement of a point at a distance r in front of the wave has an x-component

$$v_x = \int \frac{d}{dr} \left[\frac{\sigma\left(t - \frac{r}{V}\right)}{r} \right] \cos(r, x)\, dS,$$

whereas Fresnel's assumption would give

$$v_x = \int \frac{\sigma\left(t - \frac{r}{V}\right)}{r} \cos(r, x)\, dS.$$

A similar result has already been arrived at by M. Gouy, using a different line of argument.

The case of a sphere with a single source at its centre is next considered, and it is shown that the phase of the vibratory motion of an element of surface of the sphere, in order to produce the same effect at any point outside as the source at its centre does, must be independent of the radius of the sphere : it is, in fact, the phase of the centre of the sphere. In arriving at this result each part of the surface is supposed to be transparent to the action of the other parts, the velocity-potential is supposed to be continuous at the surface, and the vibratory velocity discontinuous. A method of realising this case is indicated, in which two vibrating layers are placed one on either side of the spherical surface at a very small distance apart ; in their vibration they alternately approach towards, and recede from, each other. Such a double layer corresponds to the single surface distribution in the electrical problem, and it satisfies all the conditions of Huygens' problem except that it is not a single superficial distribution of sources: the author calls its determination a Huygens' problem of the second order. The analogous electrical problem is the calculation of the charge to be given at each point of a surface to a double layer of electrified matter coincident with the surface, in order that the layers may exert the same force at all points outside the surface as a given series of charges within the surface. This problem appears to be

solvable in the general case, but it breaks down in some particular cases, such as, for example, a spherical surface with a single attracting point at its centre. There is this difference between the two orders of solution of Huygens' problem, namely, that in the case of a single layer of sources the intensity of the radiation sent from any element is the same in all directions, whereas with the double layer the intensity of radiation from any element along any line varies with the angle between that line and the normal to the surface at the position of the element.   J. L. H.

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225. *Carriage of Light-Waves by Matter in Motion.* **G. Foussereau.** (C. R. 120. pp. 85–88, 1895.)—In a previous paper (J. Phys. 1. p. 144) the author has given a proof of Fresnel's formula for the transport of light-waves by ponderable matter, which applies to any disturbance giving rise to a plane wave moving in the direction of motion of the matter. The present paper extends this proof to the case when the wave and the matter do not move in the same direction. The square of the ratio of velocity of moving matter to velocity of light is neglected in comparison with unity, and the bound ether is regarded as an inert mass, without elasticity, added to the free ether. With these assumptions it is shown that a plane wave will remain such in the moving medium, but the direction of its motion will not be perpendicular to the wave-front. The velocity of the wave relatively to the moving medium is found by compounding its velocity in the stationary medium with $\dfrac{v}{n^2}$ in a direction opposite to that of the motion of the medium, v being the velocity and n the refractive index of the medium.   J. L. H.

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226. *Huygens' Principle in Isotropic Bodies.* **E. Carvallo.** (C. R. 120. pp. 88–91, 1895.)—The assumptions made in the proofs of Huygens' principle hitherto given render it inapplicable to cases involving dispersion or double refraction. The author starts with the ordinary differential equation of light-waves in an isotropic medium, and makes use of an integral of it found in a previous paper (C. R. 118. p. 1003, 1894.) It is then shown that the volume-integral which expresses the effect of any luminous sources on an external point can be transformed into a surface-integral taken over its bounding surface σ, which separates the sources from the point. The surface σ need not be closed, provided it has infinite branches.   J. L. H.

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227. *Frequency of Absorption-Rays of Isotropic Substances.* **G. Moreau.** (C. R. 119. pp. 422–424, 1894.)—This is the sequel to a communication by the same author (C. R. p. 324, 1895). The author assumes an interaction between matter and ether, represented in the former communication by a number of small bodies

(ether) impinging on a sphere M of matter. If the small bodies have unequal principal axes of inertia, this gives rise to a certain ellipsoidal distribution; but if they be equal, as in the paper now under consideration, the body being supposed to be isotropic, the ellipsoid becomes a sphere.

The equations of motion are given in the form:—

Ether ......
$$\rho\,\frac{d^2\xi}{dt^2}=\alpha^2\left(\Delta\xi-\frac{d\theta}{dx}\right)+\beta(U-\xi)+\gamma\frac{d}{dt}(U-\xi),$$

Matter ....
$$M\frac{d^2U}{dt^2}=aU+\beta(\xi-U)+\gamma\frac{d}{dt}(\xi-U),$$

in which $\gamma = NP$, N being the density of the absorbent substance and P a constant. Also, a represents molecular action on matter; a is negative; β represents attraction between ether and matter, and is positive. Two solutions of these equations consistent with the motion being transversal are given, viz.:

$$\left.\begin{array}{l}\xi=Ae^{-Kz+in\left(\frac{z}{C}-t\right)},\\[1mm]U=Be^{-Kz+in\left(\frac{z}{C}-t\right)};\end{array}\right\}\quad\cdots\quad\cdots\quad(1)$$

$$\left.\begin{array}{l}\xi=Ae^{(\lambda-in)t+in\frac{z}{C}},\\[1mm]U=Be^{(\lambda-in)t+in\frac{z}{C}};\end{array}\right\}\quad\cdots\quad\cdots\quad(2)$$

in which A and B are independent of z and t.

From (1) is obtained

$$\frac{K}{C}=\frac{\gamma}{2\alpha^2}\frac{(Mn^2+a)^2}{p^2+n^2\gamma^2},\quad p=Mn^2+a-\beta,$$

$$\frac{1}{C^2}-\frac{K^2}{n^2}=\frac{\rho}{\alpha^2}+\frac{Mn^2+a}{n^2\alpha^2}-\frac{p\beta+n^2\gamma^2}{p^2+n^2\gamma^2}.$$

The factor γ varies as N the density of the absorbing substance, the velocity C and n. Whence it follows that γ increases rapidly with n. This gives rise to one or more absorption-bands with irregular dispersion.

The system (2) leads to a cubic equation in λ with one real negative root, lying between

$$-\gamma\frac{(Mn^2+\beta)(\rho+M)}{2\rho M^2 n^2}\quad\text{and}\quad-\gamma\frac{(p+\beta)^2}{2[\rho(p^2+n^2\gamma^2)+M(\beta^2+n^2\gamma^2)]}$$

and γ being small, this expresses a damped or slowly subsiding wave.

The index of refraction, if V_0 be velocity of light *in vacuo*, is given by

$$\frac{N^2}{V_0^2}=\frac{\rho}{\alpha^2}+\frac{p(2\rho\lambda_1-\gamma)-\beta(2M\lambda_1-\gamma)}{(2M\lambda_1-\gamma n^2\alpha)^2},$$

λ_1 being the mean value of λ for a certain part of the spectrum. This shows that, in order that the subsiding wave may be real, p must be necessarily positive, *i. e.* $Mn^2 + a - \beta$ necessarily positive, which must be the case if n the number of vibrations per second be great enough.

For small values of γ, $p=0$ corresponds to a band of irregular dispersion derived from solution (1). If that be seen in the ultra-red, the subsiding wave will have regular dispersion in the visible spectrum. The intensity is

$$I = \frac{A^2}{4} \lambda_1 X,$$

in which
$$X = \frac{\lambda_1^2 + n^2}{\lambda_1} - n \sin 2n \frac{z}{C} + \lambda_1 \cos 2n \frac{z}{C};$$

that is, I is proportional to λ_1, which is proportional to γ; so I is, in general, small.

I. X is maximum when
$$\alpha_1 + \tan^{-1}\frac{A_1}{B} - \sin^{-1}\frac{C}{\sqrt{A_1^2 + B^2}} = 2K\pi.$$

II. X is maximum when
$$\alpha_1 + \tan^{-1}\frac{A_1}{B} + \sin^{-1}\frac{C}{\sqrt{A^2 + B^2}} = 2K\pi,$$

in which K is a positive integer,
$$\alpha_1 = 2n\frac{z}{C}, \quad A_1 = n\frac{d\alpha_1}{dn}, \quad B = 1 + \lambda_1\frac{d\alpha_1}{dn}, \quad C = \frac{2n}{\lambda_1},$$

and minimum when
$$\alpha_1 + \tan^{-1}\frac{A_1}{B} + \sin^{-1}\frac{C}{\sqrt{A_1^2 + B^2}} = (2K+1)\pi.$$

III. In the neighbourhood of absorption-bands derived from (1) the maxima and minima are widened, and the absorption-bands have ill-defined edges. Further towards the violet the absorption-bands are more sharply defined, and the dark bands have sharp edge on the side towards red. Further still, towards the violet the maxima and minima disappear.

IV. The absorption-bands of the subsiding wave are accompanied by regular dispersion, and are periodic, as the author proposes to show.

The author concludes that in an isotropic absorbing substance composed of one sort of molecules we should expect to find two possible kinds of waves—the one kind giving bands of irregular dispersion capable of becoming in certain cases narrow dark bands with regular dispersion; the other giving a more numerous class of bands of less intensity.   S. H. B.

228. *Glass-Silvering Process.* **F. L. O. Wadsworth.** (Zschr. f. Instrumk. 15. pp. 22–27, 1895.)—This is a modification of the method first published by Brashear. The reducing liquid consists of sugar-candy 90 gr., nitric acid (density 1·22) 4 c.c., alcohol 175 c.c., and distilled water 1000 c.c. The alcohol and nitric acid are added after the sugar is dissolved. The liquid improves with keeping. The silver solution is an ammoniacal solution of the oxide precipitated from the nitrate. To this solution is added another of caustic potash, in the proportion of one part of potash to two of silver oxide. The quantities required for a mirror 10 cm. in diameter are 1·8 gr. AgNO$_3$, 0·9 gr. KHO, 1·5 c.c. ammonia (density 0·88), and 10 c.c. of the reducing liquid. Special care is necessary in preparing the silver solution. Silver nitrate is dissolved in water to make a one-per-cent. solution. Half the ammonia is added, and the rest is diluted with water in the proportion of 1 to 5, and is then added slowly, until the silver precipitate is just re-dissolved. During the last part of this operation the vessel must be shaken in order to wash off the solution adhering to the walls. The caustic potash solution is then added and well stirred. Any remaining precipitate is nearly re-dissolved by ammonia as before. The liquid should finally have a light brown colour, indicating a small quantity of free silver oxide. After this, the reducing liquid is added, and the whole is poured into the silvering vessel. The silvering is completed in 10 or 15 minutes at a temperature of 20° C. The mirror is washed in running water, strongly rubbed with a cotton rubber, and dried in a warm place. No further polishing is necessary. The cleaning of the surface to be silvered is best accomplished by washing in hot, strong soap-solution, rubbing with cotton, immersing in strong nitric acid, and again rubbing. The glass is then immersed successively in strong caustic potash and in distilled water, rubbing after each operation. Alcohol is superfluous and even disadvantageous.   E. E. F.

229. *Silvering Glass.* **A.** and **L. Lumière.** (J. Phys. 4. pp. 29–30, Jan. 1895.)—To 100 c.c. of 10 % silver-nitrate solution add just enough ammonia to redissolve the precipitate formed ; dilute to a litre : that is liquid A. Take commercial formaldehyde (40 % solution) and dilute it to a 1 % solution : that is liquid B. Having cleaned the glass surface, mix two volumes of A with one of B and pour at once on the glass ; in five or ten minutes all the silver is deposited, and the film may be washed with water, and varnished or burnished.   R. A. L.

230. *Light Phenomena during Crystallisation.* **E. Bandrowski.** (Zschr. phys. Chem. 15. pp. 323–326, 1894.)—It is well known that arsenious oxide, on crystallising from its hydrochloric acid solution, and the double potassium sodium sulphate and sodium fluoride from their saturated aqueous solutions, evolve light. The

cause of this phenomenon is unknown, but has been attributed to the formation of crystalline complexes of the separated molecules. The author is of the opinion that electrolytic dissociation in the solution is the origin of these appearances, and that therefore the light may be regarded as an electrical manifestation supporting this view of the molecular condition of solutions. From this hypothesis it follows that the phenomena should obtain in only solutions of salts which are capable of electrolytic dissociation. The crystallisations of $NaCl$, KCl, KBr, K_2SO_4, and KNO_3 from aqueous solutions are examined and no light-production is observed. The negative result might, however, be due to the slowness of the union of the ions under the experimental conditions. By the addition of hydrochloric acid or alcohol to a saturated aqueous solution, the crystallisation is hastened, and the union of the dissociated ions is then accompanied by light phenomenon, in the case of KCl and $NaCl$. KBr precipitated from its aqueous solution by alcohol gives a less satisfactory result, but the author believes that by modifications of the experiment he will be able to obtain the phenomenon in this case also. He is extending the investigation on these lines and those of Rose, Pogg. Ann. 35. (1835) and 52. (1841).   S. R.

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231. *Improved Dichroscope.* **G. Halle.** (Zschr. f. Instrumk. 15. p. 28, Jan. 1895.)—The dichroscope invented by Haidinger and improved by Prof. von Lang is further improved by making the opening at one end of the tube containing the Iceland spar rectangular instead of square, so that the two images seen through the lens at the other end together form a square. This gives a larger field with the same length of crystal. Over this square hole is mounted a stage with spring clamps to carry the mineral under examination, and the collar holding this stage is provided with a circle divided from 5 to 5 degrees. As the stage is made of aluminium, the whole instrument does not weigh more than 62 gr.   E. E. F.

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232. *Diathermancy of Transparent Substances.* **M. Aymonnet.** (C. R. 119. pp. 50–52, 1894.)—A thermopile is substituted for the eyepiece of a spectroscope with achromatic lenses and a flint-glass prism, an image of the source of light is thrown on the slit by means of two crown-glass lenses, and the heating effects of different parts of the visible spectrum compared. Troughs of glass, empty or filled with water or chloroform, are interposed between the source and the slit; and the author deduces for the diathermancies of these materials values which vary irregularly with the wave-length. When the temperature of the Bourbouze lamp used as the source of light is varied, the author also finds the diathermancy for rays of the same wave-length to vary very considerably.   L. R. W.

233. *Aberration observed with a Telescope full of Water.* **H. Pellat.** (J. Phys. 4. pp. 21–23, Jan. 1895.)—It is assumed (1) that the object-glass of the telescope is thin enough to bring its two principal planes within a negligible distance of each other ; and (2) that when a medium of refractive index n moves with the velocity v, the ether occupying the same place may be accounted to move with the velocity $v(n^2-1)/n^2$. From these data the author shows that, with an approximate accuracy within the errors of observation, the value of the aberration is independent of the nature of the transparent medium contained between the object-glass and its focal plane. E. H. B.

234. *Anomalous Rotatory Dispersion of Absorbent Crystals.* **G. Moreau.** (C. R. 120. pp. 258–261, 1895.)—In the equations of motion of a luminous disturbance in an absorbent medium a resisting force is introduced and expressed as a function of the displacements of the ether and of the gross matter. Solutions of the resulting differential equations are then given and discussed. The author thus accounts for (1) Unequal absorption of the right-handed and left-handed waves, (2) Absorption bands, and (3) Anomalous rotatory dispersion. E. H. B.

235. *Permanent Magnet at Low Temperatures.* **R. Pictet.** (C. R. 120. pp. 263–264, 1895.)—A small permanent magnet is placed in a cold bath of alcohol, and the attraction between it and its armature, when separated by a 3 mm. thick layer of ice, is measured by a sensitive balance. The temperatures are given for each interval of 5° from +30° to −105°. The force varies from 57·31 grammes weight at the highest temperature to 76·64 at the lowest. E. H. B.

236. *Fresnel's Biprism.* **G. Meslin.** (C. R. 120. pp. 261–263, 1895.)—The usual elementary theory of interference phenomena with Fresnel's Biprism neglects the thickness of glass traversed, and treats the two virtual images of the luminous source as true images and not as caustics. In both these respects the theory errs. A closer examination, however, confirms the result of the elementary theory, but traces the phenomena to a different source, as follows :—The retardation at a point arises, not from the *geometrical* difference of the two paths traversed by the interfering rays (for, neglecting the paths in the glass, this difference is always zero), but is due simply to the *different thicknesses of glass* passed through in the two cases. This view was experimentally verified. E. H. B.

237. *Colours of Iodine Solutions.* **E. Thiele.** (Zschr. phys. Chem. 16. pp. 147–155, 1895.)—This paper describes spectroscopic and photometric observations of iodine solutions undertaken in the hope of ascertaining the cause of their different colours. Various

, solvents are employed, and with each one twenty degrees of con-
centration are adopted, ranging from 0·05 to 0·00001 gm. of iodine
per c. c. The absorption ratio is thus found to be not a constant
quantity. With all the solutions used it falls off steadily for a time
as the dilution is continued, but, in some cases, it afterwards rises
suddenly with the weakest solutions. Experiments are also made
in which the two parts of a beam of light pass through the same
quantity of iodine and contained in the same solvent. The upper
half of the beam, however, passes through a layer 1 mm. thick of
concentrated solution, while the lower half passes through a layer
1 m. thick of proportionately diluted solution. With this arrange-
ment no sensible displacement is observable in the absorption-bands,
but they are more strongly marked in the stronger solutions.

E. H. B.

238. *Phosphorescence at Low Temperatures.* **R. Pictet** and
M. Altschul. (Zschr. phys. Chem. 15. pp. 386–388, 1894.)—
Glass tubes containing phosphorescent substances, like the sulphides
of calcium, strontium, and barium, were exposed to sunlight for one
minute and then placed in an enclosure reduced to a temperature
of −140° by a jacket of liquid nitrous oxide boiling into a vacuum.
After 12 minutes the tubes were quickly transferred to a dark
chamber. At first no trace of light could be seen, but phos-
phorescence soon appeared, and after five minutes reached its
normal intensity, although the tubes had not been exposed to
light since being cooled. In order to determine at what tem-
perature the phosphorescence ceased, alcohol was cooled to −80°,
and the tubes partially immersed in it after being exposed to sun-
light. The immersed portion ceased to phosphoresce when its
temperature sank to −65°; the upper portion, which was not in
the alcohol, continued to phosphoresce strongly. Before the light
disappeared it assumed a pale yellow colour. The results are in
accordance with the idea that phosphorescence is due to the mole-
cular vibrations of the substances. W. C. D. W.

HEAT.

239. *Radiation at Low Temperatures.* **Raoul Pictet.** (C. R. 119. pp. 1202–1206, 1894.)—Refrigerating cylinders of various sizes and shapes were cooled to temperatures between − 100° and −170°. They were then allowed to resume the temperature of the laboratory by radiation and conduction through their surfaces. The latter were left bare, or protected by layers of cotton-wool of various thicknesses up to 50 cm. It was found that between − 170° and −100° the amount of radiation per unit of surface-area was independent of the presence or thickness of the protecting layer, and the rise of temperature was much more rapid than would be given by Newton's law. It assumed a more normal character from −70° upwards, and the influence of the covering began to make itself felt. Coverings of wool, silk, powdered charcoal, sawdust, and cork gave similar results. For radiation corresponding to very low temperatures they are practically diathermanous, and they revert to their normal condition for radiation corresponding to −70° and upwards. Their own temperature does not seem to influence this property.   E. E. F.

240. *Properties of Bodies at the Absolute Zero.* **C. M. Guldberg.** (Zschr. phys. Chem. 16. pp. 1–17, 1895.)—From the characteristic equation

$$\frac{p}{Rt} = \frac{1}{v - v_0} - f(v \cdot t)$$

the author deduces five methods of calculating the limiting volume below which the volume of a body cannot be reduced. The approximations and assumptions made in the course of the work are some of them admittedly of doubtful validity, and are too long to reproduce. As an instance of the kind of agreement obtained, the following are the values found for the limiting molecular volume of ether by the different methods :—

$$76 \cdot 9, \quad 76 \cdot 5, \quad 76 \cdot 6, \quad 89 \cdot 2, \quad 75 \cdot 9.$$

The author further obtains two discordant expressions for the relation of the critical volume to the limiting volume, and also gives expressions for the compressibility, the latent heat, and the specific heat at the absolute zero.   J. W. C.

ELECTRICITY.

241. *Application of the Principle of Least Constraint to Electro-dynamics.* **A. Wassmuth.** (Sitzungsberichte der math.-physik. Classe d. k. bayer. Akad. Band 24. Heft 2.)—This is an application of the method employed by Boltzmann, in his ' Vorlesungen,' for deducing the known equations of electrodynamics from Lagrange's equations. The author here starts from the principle of Least Constraint. That principle, as applied to continuous motions, asserts that for a system of n degrees of freedom subject to r constraints in the form $\phi_1 = 0 \ldots \phi_r = 0$, where ϕ_1 &c. are functions of the coordinates,

$$Z = \Sigma m \left[\left(\frac{d^2 x}{dt^2} - X \right)^2 + \left(\frac{d^2 y}{dt^2} - Y \right)^2 + \left(\frac{d^2 z}{dt^2} - Z \right)^2 \right]$$

must be minimum subject to the conditions ϕ. That is, we obtain the equation

$$m(\ddot{x} - X) + \lambda_1 \frac{d\phi_1}{dx} + \ldots + \lambda_r \frac{d\phi_r}{dx} = 0,$$

where the λ's are indeterminate multipliers. This equation, if there be no ϕ's, resolves itself into the ordinary equations of motion. Expressing the same thing in generalised coordinates, let the kinetic energy of the system be

$$T = \tfrac{1}{2} \{ a_{11} \dot{p}_1{}^2 + a_{12} \dot{p}_1 \dot{p}_2 + \ldots \}.$$

Let Δ denote the determinant of the coefficients a; Δ_{11}, Δ_{12}, &c. its minors. Let the Lagrangian equations be

$$T_1 - P_1 = 0, \quad T_2 - P_2 = 0, \quad \&c.,$$

in which

$$T_1 = \frac{d}{dt} \frac{dT}{dp_1} - \frac{dT}{dp_1}, \quad \&c.$$

Then, using a transformation given by Lipschitz, it is shown that by the principle of Least Constraint the following function must be minimum, viz. :—

$$Z = \frac{1}{\Delta} \left\{ \begin{array}{l} \Delta_{11}(T_1 - P_1)^2 + \Delta_{22}(T_2 - P_2)^2 + \&c. \\ + 2\Delta_{12}(T_1 - P_1)(T_2 - P_2) + \&c. \end{array} \right\} \quad \cdot \quad \cdot \quad \cdot \quad \text{(I.)}$$

That is, for each $\ddot{p}$, $\dfrac{dZ}{d\ddot{p}} = 0$, given p and $\dot{p}$. It is shown that this leads to $T_1 - P_1 = 0$ &c., that is, to Lagrange's equations. Several examples are given. For instance, the case of two cyclic coordinates $\dot{p}_1 = l_1'$ and $\dot{p}_2 = l_2'$, in the notation of the Vorlesungen. We have here

$$T = \frac{A}{2}l_1'^2 + \frac{B}{2}l_2'^2 + C\,l_1'\,l_2',$$

$$T_1 = \frac{d}{dt}(A\,l_1' + C\,l_2'),$$

$$T_2 = \frac{d}{dt}(B\,l_2' + C\,l_1').$$

Assume impressed forces L_1 and L_2, and Lord Rayleigh's dissipation function

$$F = \tfrac{1}{2}b_{11}l_1'^2 + \tfrac{1}{2}b_{22}l_2'^2.$$

Then the equation (I.) assumes the form

$$Z(AB - C^2) = B\left[\frac{d}{dt}(A\,l_1' + C\,l_2') - L_1 - b_{11}l_1'\right]^2$$
$$-2C\left[\frac{d}{dt}(A\,l_1' + C\,l_2') - L_1 - b_{11}l_1'\right]\left[\frac{d}{dt}(B\,l_2' + C\,l_1') - L_2 - b_{22}l_2'\right]$$
$$+A\left[\frac{d}{dt}(B\,l_2' + C\,l_1') - L_2 - b_{22}l_2'\right]^2,$$

from which by differentiation we obtain the usual equations for two closed circuits with resistances b_{11} and b_{22}.　　　　S. H. B.

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242. *Measurement of Resistances by means of Alternating Currents and the Telephone.* **R. Colson.** (C. R. 119. pp. 1261–1263, 1894.)—In measuring inductionless and nonpolarising resistances with a Wheatstone bridge, using a telephone and alternating currents produced by an induction coil, the relation between the four branches giving a balance rests on Ohm's law, that is to say, on the fall of potential being proportional to the resistance. This law is, however, no longer admissible under certain conditions for the propagation of waves of high potential in considerable resistances, such as those of threads soaked in saline solutions or capillary tubes containing water. Call the points of the bridge quadrilateral where the exciting source is applied A and B, and those to which the telephone is connected C and D, the telephone itself T. The potential at each of the points C and D results from two waves of contrary sign coming from A and B in opposite directions. If the point C, for instance, be such that these two waves pass it at the same time, with potentials equal in absolute value, it has zero potential and is called a neutral point; then a telephone, one terminal of which is in contact with C and the other insulated, gives no sound. One condition for measurement is to make the resistances AC and CB such that C is a neutral point, and the same applies to D, as there will only be real silence when the conditions of C and D are identical. The wave which, setting out from A, has traversed ACB, sends into BD a disturbance, which complicates that which passes in D. Break the connections CT,

AD, and that of B with the source, then connect B with a coil identical with that of the source so as not to affect capacity. If the telephone be silent, it shows that the variations of potential at D coming by the path ACBDT are too weak to produce the trouble in question. If the telephone give a sound, either increase the resistances ACB or diminish the potential at A. Test in the same manner for the other three paths. It is also necessary to find out if the wave which returns to C after having traversed the whole circuit bring with it any complication. To do this, connect to C a resistance equal to that of the whole circuit and connect with one terminal of the telephone, if there be any sound increase the resistances or diminish the potential. E. C. R.

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243. *Some Values of the Magnetic Elements for Jan. 1st, 1895.* **T. Moureaux.** (C. R. 120. pp. 42–43, 1895.)—The values given below are deduced from the hourly readings of the magnetograph curves for Dec. 31, 1894, and Jan. 1 & 2, 1895. The values of the ordinates of the several curves are deduced from absolute determinations made on Dec. 27 & 31, 1894, at Parc Saint-Maur, and on Dec. 29 and Jan. 2 at Perpignan, these days being free from magnetic disturbances. The values of the secular change are obtained from a similar series of numbers for Jan. 1st, 1894, given previously (C. R. vol. cxviii. p. 70, 1894).

Parc Saint-Maur.   Long. 0° 9′ 23″ E.; Lat. 48° 48′ 34″ N.

	Value of element on Jan. 1st, 1895.	Secular change.
Declination	15° 12′·7	−5′·3
Dip	65° 4′·9	−1′·2
Horizontal component	0·19641	+0·00017
Vertical component........	0·42277	−0·00003
Total force	0·46617	+0·00005

Perpignan.   Long. 0° 32′ 45″ E.; Lat. 42° 42′ 8″ N.

	Value of element on Jan. 1st, 1895.	Secular change.
Declination	14° 3′·4	− 5′·0
Dip	60° 9′·9	−0′·8
Horizontal component	0·22345	+0·00025
Vertical component........	0·38951	+0·00021
Total force	0·44914	+0·00031

W. W.

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244. *Law of Transmission of Energy between Source and Conductor in the Case of a Permanent Current.* **M. Vaschy.** (C. R. 120. pp. 80–82, 1895.)—When a permanent current is maintained in a conducting circuit, the energy dissipated in each unit of volume can be calculated by Joule's law. This energy is constantly restored to the volume from an external source, so that the flux of energy into the volume is exactly equal to the rate of its dissipation.

The author obtains from this an equation of continuity of energy, expressing the magnitude and direction of the flux of energy at any point by the components of a vector. Since the energy never penetrates into the interior of a body with an equipotential surface, such as an insulated conductor, whereas it does so when the surface is not equipotential, as in the case of a conductor carrying a current, it follows that the flux of energy must always take place along the equipotential surfaces. This condition gives a second equation to be satisfied by the flux-vector. It is then shown that the electrical state of any point in the field must depend not only on the electrostatic state (h), but also on the current, since a transmission of energy takes place between the source and the point where the energy of the source is dissipated. The field due to the current is represented by another vector (h'), and the author shows that, if the flux of energy is dependent only on h and h' at all points, it must be perpendicular to both vectors and equal to $1/4\pi$ times their product. Comparing this result with that of Poynting, obtained in an entirely different manner, we see that h' is the magnetic field of the current, which has, therefore, a purely electrical origin. J. L. H.

245. *Production of Kathode Rays.* **J. de Kowalski.** (C. R. ·120. pp. 82–85, 1895.)—The author refers to an experiment of Goldstein in which an electric discharge is passed through a vacuum tube, a part of which is constricted. Under these circumstances, the kathode rays are seen emanating from the end of the con-striction near the kathode. In the present paper a tube is described consisting of two large cylindrical bulbs joined by a capillary tube, the electrodes being in the bulbs ; when a discharge passes the capillary portion becomes intensely luminous, and rays possessing all the properties of kathode rays emerge from *each* end of it. This suggested the production of similar rays in electrodeless tubes, the discharges being set up by Tesla currents. For this purpose a tube similar in shape to the ordinary spectroscopic ·vacuum-tube was used, but having no electrodes. On setting up electric oscillations in it the rays emerging from each end of the ·constricted portion were found to have opposite properties ; one was attracted, and the other repelled, by a given magnet-pole. The production of kathode rays is therefore not confined to tubes containing electrodes : they occur whenever the current-density is sufficiently great, and they are propagated in the direction of the lines of flow of the current from the positive towards the negative pole. J. L. H.

246. *Electric Logs.* (Rev. Int. de l'Électr. 9. pp. 21–27, 1895.) —Describes the chief electric logs that have appeared during the last twenty years. The principle on which they all work is that during the passage through the water an axle is rotated by means of either screw-propellor vanes, or by cups fixed on

arms, as in an anemometer. This axle closes a circuit once every revolution, and gives a stroke on an electric bell; these strokes being counted during a given space of time, the speed of the ship can be determined. The contact-closing apparatus is either enclosed in a watertight casing, in which case two wires are employed stranded in the line by which the log is towed astern; or, as is more often the case, the axle has an insulating collar on which is a single piece of metal in connection with the whole metal-work of the log, and a single wire in the towing line is connected to a spring covered with insulating compound all but its extreme point, and the circuit is closed by the sea. The resistance is high and the current too feeble to ring the bell except when the spring is in contact with the metal piece, when the surface in contact with the sea being much increased, a sufficient current flows to make the bell sound. The latest log described is that of Admiral Fleuriais, who employs the latter method of working the bell, and an anemometer wheel consisting of four hemispherical cups attached to the ends of radial spokes. In his latest pattern he uses one of these wheels at each end of the axle, in the middle of which is an endless screw working a cogged wheel, on whose axle is an insulated collar with three contacts. There are also two curved vanes attached to the apparatus to ensure proper immersion.   E. C. R.

247. *Aid to Victims of Electric Shocks.* (Rev. Int. de l'Électr. 9. pp. 27–28, 1895.)—A commission of the Academy of Medicine composed of MM. Bouchard, d'Arsonval, Laborde, and Gariel, have issued instructions for the treatment of persons who have been subjected to electric shocks, which are summarised as follows:— If the contact still exists, stop the generating machine, cut the conductor with insulated tools, or make a path to earth by means of a conductor of low resistance which will diminish the current in the part where the victim is in contact with the main conductor. Place the victim in an airy place, and avoid many crowding round. Take off the clothes, and try and establish respiration and circulation as quickly as possible. To establish respiration have recourse to the two following methods:—(1) *Rhythmical traction of the tongue.* Open the mouth, firmly grasp the fore part of the tongue between the finger and thumb of the right hand, and subject it to strong and repeated rhythmical pullings followed by relaxations of at least 20 a minute, imitating the rhythmical movement of respiration. This should be unceasingly done for a half hour or more. (2) *Artificial respiration.* Lay the victim on his back, the shoulders slightly raised, the mouth open, the tongue quite free. Grasp the arms at the elbows, and press them on to the chest, separate them and carry them above the head, making them describe the arc of a circle, finally bringing them back to their original position. Repeat these movements about 20 times a minute. It is best to commence with the traction of the tongue, using at the same time, if possible, the artificial respiration, also trying to restore circula-

tion by rubbing the surface of the body and beating it with the hands or moistened cloths, by throwing cold water from time to time on the face, and by holding ammonia or vinegar under the nose. E. C. R.

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248. *Secular Variation of Terrestrial Magnetism.* **L. A. Bauer.** (Inaugural Dissertation : Berlin, 1895.)—The author has calculated and drawn the paths which the north-seeking poles of magnetic needles freely suspended at a number of different places would have traced in consequence of the secular change. The important conclusion is drawn that to an observer placed at the centre of the needle, the north pole would in all cases have appeared to describe its orbit in the same direction as the hands of a watch. This generalisation is verified for 23 places, distributed through both hemispheres. None of the curves are as yet sufficiently complete to enable an opinion to be formed as to whether they will be re-entering. A. W. R.

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249. *Theory of the Influence Machine with oppositely rotating Discs.* **W. Holtz.** (Wied. Ann. 54. pp. 181–190, 1895.)—As a preliminary, the case of two infinitely long strips of insulating material travelling in opposite directions is considered—these with two inductors affording an elementary form of machine. Here equal quantities of opposite electricity travel on the strips in opposite directions to infinity. Next, by substituting disks these opposite charges are brought round towards each other and can be mutually neutralised by the aid of connected inductors. Step by step the various kinds of machines possible are evolved from the elementary form. The author shows that for the successful work-ing of a machine the conditions involved in what he calls the principle of reinforcement must be observed. He states that he has in former papers (1867–1881) published much of the results given in this paper, and claims to have anticipated Mr. Wimshurst with reference to the machine associated with his name in England by two years. F. L. T.

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250. *Formation of Ozone by Electrical Oscillations.* **E. Wiede-mann** and **G. C. Schmidt.** (Wied. Ann. 53. pp. 924–927, 1894.)—Oxygen under a pressure of less than 9 mm. is slowly passed through the space between two concentric tubes, coated internally and externally with tin-foil used as electrodes for alternating currents of constant frequency obtained by a Lecher's apparatus. The authors conclude from their observations that regular oscillations of a *certain* frequency are more efficient for the formation of ozone than irregular oscillations. A. G.

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251. *Self-Induction of Iron Wires.* **I. Klemenčič.** (Wied. Ann. 53. pp. 1053–1061, 1894.)—The paper deals mainly with the experimental determination, by means of the well-known

method with Wheatstone's bridge and galvanometer, of the coeffi-
cients of self-induction of three pieces of wire, made respectively
of soft iron, hard iron, and Bessemer steel, each 1 metre long and
2 mm. diam.   The table below indicates the values, S, of the
coefficient of self-induction for various currents, i—both in absolute
measure.   V is the ratio of the actual self-induction to that which
the wire would have if it were not magnetic.

Soft iron.			Hard iron.			Bessemer steel.		
i	S	V	i	S	V	i	S	V
0·012	10,890	9·2	0·013	5,330	4·5	0·011	4,320	3·7
0·021	13,480	11·4	0·022	6,140	5·2	0·021	4,720	4·0
0·043	15,340	13·0	0·044	8,340	7·1	0·042	4,820	4·1
0·079	21,290	18·0	0·082	9,690	8·2	0·077	5,120	4·3
0·116	31,760	26·9	0·121	11,840	10·0	0·111	5,460	4·6
0·134	37,770	32·0	0·139	12,730	10·8	0·127	5,340	4·5
0·218	72,090	61·1	0·226	23,810	20·2	0·201	5,970	5·1
0·322	79,060	67·0	0·334	39,870	34·0	0·292	6,193	5·2

From the previous determinations are calculated the permeabili-
ties of the various specimens for circular magnetisation, and these
are compared with the permeabilities of the same specimens for
axial magnetisation.   In the following table, H_m represents the
average field caused in the mass of the wire by the current under
consideration, H the magnetic field when the wire is magnetised
longitudinally, and k the permeability.   In each line of the fourth
column two values of k are given; the first is that which was
obtained *before* the wire had been circularly magnetised, &c. ; the
second is that obtained *after*.

Soft iron.				Hard iron.				Bessemer steel.			
Circular.		Longitudinal.		Circular.		Longitudinal		Circular.		Longitudinal.	
H_m	k	H	k	H_m	k	H	k	H_m	k	H	k
0·16	16	0·12	23 —17	0·17	8	0·12	7	0·15	6·9	0·12	6·2
0·27	20	0·31	26—21	0·29	9	0·31	9	0·28	7·6	0·31	5·9
0·54	23	0·63	32—27	0·60	13	0·62	10	0·57	7·7	0·58	6·0
1·00	32	0·95	39—32	1·10	14	0·96	12	1·03	8·2	0·94	6·2
1·48	48	1·54	61 —46	1·63	18	1·54	15	1·49	8·7	1·55	6·2
1·71	57	1·85	85—61	1·88	19	1·83	16	1·71	8·6	1·84	6·3
2·76	108	2·91	173—143	3·04	36	2·91	25	2·70	9·6	2·90	6·6
4·1	118	4·45	177 —155	4·49	60	4·44	41	3·93	9·9	4·43	7·1

A. G.

252. *Electrification derived from Red-hot Conductors.* **G. Vi-**
centini and **M. Cinelli.**   (N. Cim. 36. pp. 297–313, Dec. 1894.)
—A glass globe with four mouths is used to measure the electrifi-
cation of a metallic plate derived from an incandescent wire across

air or other gas. The two lateral mouths serve to admit thick copper wires connected by a piece of platinum wire 0·15 mm. thick at the centre of the globe. Through the top mouth a glass tube enters the globe, containing a platinum wire bearing a small plate of platinum at its lower end, fixed at a distance of 10 cm. above the horizontal wire. The plate is connected with an electrometer. The globe may be filled with various gases through the upper or lower mouths. On heating the horizontal wire to a red heat by means of a current, the plate assumes potentials slightly higher than the mean potential of the glowing wire, the excess varying with the gas employed. The potential decreases when the plate is raised, but this change is very slight when the distance is about 10 cm. On mounting the plate below the wire the potential rapidly falls with increasing distance, almost vanishing at 12 cm. The potential acquired also depends upon the temperature of the glowing wire. This is measured by the resistance, from the known temperature-coefficient of platinum. The resistances at which maximum electrification was observed were 0·65 ohm in hydrogen, 0·73 ohm in air, and 0·75 in carbonic acid, so that the three gases require correspondingly increasing temperatures to convey their maximum charges. The average potential of the incandescent wire was 4 volts, the maximum potential acquired by the plate by convection through hydrogen was 4·25 volts, and through air and carbonic acid 5 volts.   E. E. F.

253. *Magnetic Effects of " Extra-Current."* **Carl Fromme.** (Wied. Ann. 54. pp. 1–29, 1895.)—The author continues his investigations upon the effect of introducing a condenser between the terminals of the interrupter of a coil ; with special reference to the behaviour of the iron within the coil, and the rate at which its magnetisation changes with various capacities and resistances added to the coil or condenser circuits. Tests are made upon bundles of iron wire in oil or mercury, and steel rods, for cores. The resistances used were by different makers, and their relative effects are compared. Details of the methods employed for these measurements do not appear in the paper.   R. A.

254. *Calculation and Measurement of Small Self-Inductions.* **M. Wien.** (Wied. Ann. 53. pp. 928–947, 1894.)—Maxwell's principle of " geometric mean distance " for the calculation of self-induction is applied not only to straight but also to *curved* wires of various sections ; it is shown by a more thorough mathematical determination of the self-induction of a ring that the error, in this particular case, is only a fraction of a per cent. Measurements by means of a method described by Prerauer (Wied. Ann. 1894), which involves the use of a Wheatstone's bridge, telephone, and alternating current, confirm the accuracy of the calculations. Finally there are experiments on the self-induction of iron and steel wires for various strengths of alternating current.   A. G.

255. *Conductivity of Aqueous Solutions of Carbonic Acid.* **W. F. Knox.** (Wied. Ann. 54. pp. 44–57, 1895.)—In a paper by E. Pfeiffer on the electric conductivity of carbonic acid and on a method of measuring the resistance of liquids under high pressures (Wied. Ann. 23. p. 625, 1884), an account is given of researches on the conductivity of solutions of carbonic acid in water at 0° and 12° 5 under pressures varying from 1 to 25 atmospheres. As carbonic acid is generally present in water, a knowledge of the properties of its dilute solutions is also of great interest. The author has therefore investigated the conductivity of solutions of carbonic acid in water under pressures from $\frac{1}{40}$ of an atmosphere to 2 atmospheres. The measurements were made with a telephone and alternating currents by Kohlrausch's method. The results for varying pressures at mean temperatures of 12°·5 and 18° are given in tabular form. These can be combined with Pfeiffer's results so as to give the whole course of the curve from a pressure of 17 mm. of mercury up to 25 atmospheres. Kohlrausch has pointed out that in the case of dilute solutions of acetic acid and ammonia, the conductivity k is approximately proportional to the square root of the amount m of substance in the solution (m being expressed in equivalents per litre). The author examines whether this holds good for dilute solutions of carbonic acid, *i. e.*, whether $k/\sqrt{m} = c$ is a constant. In the case of very dilute solutions of substances which have not a neutral reaction, it is well known that there is an uncertainty as to the conductivity of the water itself. If an allowance is made for the conductivity of the water, the values of c diminish as the dilution increases. If no deduction is made for the conductivity of the water, the values of c increase with the dilution. But within such limits that the conductivity of the water is small compared with the total conductivity, the law of proportionality with $\sqrt{m}$ is fairly confirmed.   D. E. J.

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256. *Electromagnetic Attraction.* **M. Weber.** (Wied. Ann. 54. pp. 30–43, 1895.)—For the purpose of finding the attractive force between a coil and various samples of iron, for different field-strengths, and with increasing and decreasing cycles of magnetisation, the samples, in the form of long straight wires, are suspended horizontally by two threads : one end of the wire projects, axially, into a helix carrying the magnetising current, which attracts it in the direction of its length, through a certain distance, which is observed by means of a reading microscope. The attractive force, (ρ), in dynes per cm.² of iron section, is deduced from these readings, by calculation from the known dimensions and weight of the suspended system. The intensity of magnetisation is derived from ρ, and the value of H within the helix, for each value of the magnetising current, by the formula $\rho = \mathrm{IH} = \chi \cdot \mathrm{H}^2$; results being checked by magnetometric measurements on the same sample. Some further tests with similar apparatus relate to the determination (ρ_1), the attractive force (diamagnetic) at right

angles to the direction of ρ; and a table is given showing the ratio of ρ to ρ_1 for different values of H. This ratio rapidly diminishes as H increases, tending, in the limit, towards unity.

R. A.

257. *Influence of other Electrolytes on the Conductivity of Acetic Acid.* **A. J. Wakeman.** (Zschr. phys. Chem. 15. pp. 159-182, 1894.)—The deviations from the expected results of measurements of the conductivity of solutions of organic acids led the author to investigate the effect of impurity by mixing known amounts of other bodies with solutions of acetic acid. By applying Arrhenius' theory of isohydric solutions to such a case, the author calculates the theoretical effect, and this agrees well with a long series of determinations of the conductivity of such mixtures. The agreement with theory when hydrochloric acid is added is not as good as in the cases of weaker acids, and even when it is only present in the proportion of 1 : 1000 it has a marked influence on the conductivity of acetic acid. W. C. D. W.

258. *Double Refraction of Electric Rays.* **K. Mack.** (Wied. Ann. 54. pp. 342-351, 1895.)—An arrangement analogous to the introduction of a doubly refracting crystal between two crossed Nicols was made, the Nicols being represented by Hertzian concave mirrors with their focal lines at right angles to each other, and a wooden block being substituted for the crystal. An illumination of the spark-gap of the receiving mirror will, with this arrangement, prove the existence of double refraction in the wood. The author used a deal block in the shape of a regular octagon with a distance of 60 cm. between two opposite sides : its thickness was 20 cm. When the focal lines of the mirrors were parallel, at a distance of 3 m., the rays were transmitted whatever the direction of the fibre, but they were feebler when the fibre stood vertical. When the mirrors were crossed, and the block placed with the direction of the fibre at 45° to both focal lines, and with the octagonal faces towards the mirrors, the spark-gap was illuminated, the sparks disappearing on removing the wood, or turning it through 45° in one or the other direction. With lesser thicknesses of wood the phenomenon decreased in intensity, disappearing altogether at a thickness of 5 cm. It was not sensibly diminished on gradually increasing the thickness to 40 cm. It appears from these and other experiments that one of the components of a vibration—that oscillating along the fibre—is absorbed, such absorption being complete with a thickness of 35 cm. Plates of oak and beech showed the same phenomena, the former somewhat feebler. Plates cut across the fibre showed no double refraction, as they should not by the optical analogy. The existing sparks were produced by an influence machine, connected with the Hertz apparatus in the manner described by Töpler. E. E. F.

259. *Nature of Disturbances of Telephone Wires in the Neighbour-hood of Large Alternating Currents.* **M. Pierard.** (Ecl. El. 2. 1. pp. 121–123, 1895.)—The author performs a series of experiments on telephone-wires earthed in various ways near a conductor carrying a large variable current, with a view to ascertaining whether disturbances in the telephone-wire are due to leakage-currents or currents induced electromagnetically or electro-statically. He arrives at the following conclusions :—(1) Electro-static effects are perceived first, even at distances of 50 to 300 metres, depending on the lengths of the influencing and the influenced circuits ; (2) These effects can alone cause serious hindrance to telephonic communications ; (3) When the telephone-wire is earthed near an earthed conductor carrying a large current, leakage-currents of varying magnitudes occur ; (4) Leakage-currents preponderate when the telephone-wire is earthed twice in the neighbourhood of the variable current ; (5) In comparison with other effects, those due to electromagnetic induction are in all cases negligible. W. G. R.

260. *Electric Waves generated by two Rectangular Oscillations or by a Uniform Rotation.* **A. Righi.** (R. Accad. Sci. Bologna, 11 Nov., 1894).—This is a mathematical paper. Starting with Hertz's equations of the electromagnetic field, the author obtains expressions for the electric and magnetic disturbances at any point in space due to a small rectilinear electric oscillation at the origin of coordinates and directed along the axis of z. The analogous expressions are next obtained for the disturbances due to another rectilinear oscillation of the same period and amplitude, but directed along the axis of y and differing in phase by a quarter of a period from the first. These two oscillations are then supposed to occur simultaneously, the corresponding disturbances com-pounded and their resultant fully determined as a function of the time, space-coordinates, and constants concerned. Electric charges are next supposed to describe with uniform velocities a circular orbit whose centre is at the origin, and whose plane is that of yx, the radius being equal to the amplitudes of the former rectilinear oscillations ; and expressions for the disturbances due to this are obtained. The equations for the disturbances due (1) to the two rectangular oscillations and (2) to the circular motion are each simplified and found, under certain limitations, to be identical. The disturbance due to the circular motion of electric charges is shown to be a transverse wave-motion radiating spherically from the source with amplitude inversely as the distance traversed, and therefore with intensity inversely as the square of that distance. This radiation is in general elliptically polarised, but along the axis of x it is circularly polarised, and in the plane of yz plane polarised. The probability of experimentally realising and detect-ing such waves is briefly discussed. E. H. B.

261. *The Dimensions of Magnetic Potential.* **F. Bedell.** (Phys. Rev. 2. pp. 298–302, Jan.–Feb. 1895.)—The author discusses the dimensions of magnetic potential derived from the equivalence of a magnetic shell and a circuit carrying an electric current. He observes that the dimensions of magnetic potential in this way agree in the electromagnetic system with those obtained in the usual way; but in the electrostatic system the dimensions obtained in this way are $M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}$, whereas, following Maxwell, they are $M^{\frac{1}{2}}L^{\frac{1}{2}}$. After referring to the writings of Maxwell, Clausius, and others on the subject, recognising the necessity of taking the permeability of the medium into account, he adds the following:—If V represents potential, m pole strength, r a distance, I a current, Ω a solid angle, the dimensions of V in the electromagnetic system obtained from the equations

$$V = \frac{m}{r} \quad \text{and} \quad V = I\Omega$$

are identical, because in this system of units the dimensions of the permeability, μ, are $M^{0}L^{0}T^{0}$; in the electrostatic system, however, the dimensions of μ are $L^{-2}T^{2}$, and consequently must be taken into account when considering dimensions. Let μ_e be the permeability of the space exterior to the shell, and μ that of the space occupied by the shell, l the thickness of the shell, A its area; then

$$dV = \frac{1}{\mu_e} \cdot \frac{l\,dm\,\cos\alpha}{r^2},$$

where α is the angle between the radius vector to a point P and the direction of the magnetic axis, and r is the length of the radius vector. Also

$$l\,dm = \mu_i I\,dA,$$

whence

$$dV = \frac{\mu_i}{\mu_e} I\,d\Omega,$$

and, integrating,

$$V = \frac{\mu_i}{\mu_e} I\Omega.$$

In cases usually considered $\mu_i = \mu_e$, and the last equation becomes

$$V = I\Omega,$$

and is independent of the permeability of the medium.   W. G. R.

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262. *Residual Charges of Condensers and their Dependence upon Temperature.* **F. Bedell** and **C. Kinsley.** (Phys. Rev. 2. pp. 170–196, Nov.–Dec. 1894.)—The authors first describe a series of experiments on condensers performed with a view to determining the effect of previous charges upon successive residual discharges. They first observe successive residual discharges

of a condenser after being charged positively for a given length of time; they then subject a condenser first to a negative charge for a certain time and afterwards to a positive charge, and compare the residual discharges in the two cases; they observe that in a succession of residual discharges from the same condenser the final charge at first predominates, but afterwards the previous negative charge may have the greater effect. In this way they show that the condition of the dielectric of a condenser depends upon its previous history, that is, upon the duration and signs of the various charges to which it has been subjected. They find also that the form of the discharge-curve depends upon the previous history of the condenser; that is, it is not legitimate to assume that the discharge of a condenser through its own dielectric follows the exponential law

$$V_t = V_0 e^{-\frac{t}{RC}},$$

where the symbols have their usual meanings, unless the dielectric is in a neutral condition: it is also observed that absorption has considerable effect in increasing the insulation resistance of the dielectric, but that the average resistance is practically independent of the initial potential to which the condenser is charged. In these experiments all the dielectrics are solid.

Experiments are then described which determine the effects of temperature on the resistance of various oils: condensers are discharged through the oils at different temperatures, and the authors conclude that the resistance of pure oils is a single-valued function of the temperature, and consequently that there should be no absorption of charge and that the curves for the discharge of a condenser through such oils should follow the exponential law. By observing the residual discharges and the insulation-resistance at different temperatures, it is found that the effects of absorption in solid dielectrics are diminished as the temperature is increased.

<div align="right">W. G. R.</div>

263. *Theory of Electrical and Optical Phenomena in connection with Moving Bodies.* **H. A. Lorentz.** (Leiden, 1895.)—This essay deals mainly with the question of the electromagnetic action due to the motion of charged bodies, and especially with the hypothesis of ionic charges, and applies the results obtained to the discussion of recent experiments and views with respect to the relative motion of the ether and ordinary matter. The notation adopted is to some extent similar to that of Heaviside and other vector analysts, though as a matter of fact the vector analysis itself is little used. (For clearness the German characters which the author uses for directed quantities are here translated into Clarendon type.) Thus A_l denotes the component of **A** in the direction l, so that A_x, A_y, A_z are the components in the direction of the axes of x, y, z. In the sense usually given to curl **A**, the

author uses the notation Rot $\mathbf{A}$; we shall in this abstract use the former notation. Thus

$$[\text{curl } \mathbf{A}]_x = \frac{\partial \mathbf{A}_z}{\partial y} - \frac{\partial \mathbf{A}_y}{\partial z}, \&c.$$

Again, the author writes

$$\text{div. } \mathbf{A} + \frac{\partial \mathbf{A}_x}{\partial x} + \frac{\partial \mathbf{A}_y}{\partial y} + \frac{\partial \mathbf{A}_z}{\partial z}.$$

There are then the analytical theorems—(1) for a closed surface, using $d\tau$ for an element of the volume enclosed by the surface, and $d\sigma$ for an element of the surface,

$$\int \mathbf{A}_n d\sigma = \int \text{div. } \mathbf{A} . d\tau,$$

where $\mathbf{A}_n$ is the component of $\mathbf{A}$ in the direction of the outward drawn normal, and the integrals are taken over the surface and throughout the volume : (2) for a closed curve forming the boundary of an unclosed surface, if ds is an element of the curve, and $\mathbf{B}_n$ the normal component of the curl at any point, outward from the positive side of the surface,

$$\int \mathbf{A}_s ds = \int \mathbf{B}_n d\sigma,$$

the integrals being taken round the curve and over the surface respectively.

Further, according to usage in this country we put

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2,$$

and consequently have

$$\text{curl curl } \mathbf{A} = \frac{\partial}{\partial x} \text{ div. } \mathbf{A} - \nabla^2 \mathbf{A}_x, \&c.$$

Also the product of the directed quantities $\mathbf{A}, \mathbf{B}$ is written $[\mathbf{AB}]$, and the components of the product along x, y, z are $\mathbf{A}_y \mathbf{B}_z - \mathbf{A}_z \mathbf{B}_y$, &c. The equation $\mathbf{A} = \mathbf{B}$ is to be understood as indicating that $\mathbf{A}_x = \mathbf{B}_x$, $\mathbf{A}_y = \mathbf{B}_y$, $\mathbf{A}_z = \mathbf{B}_z$, that is, that the quantities agree not only in magnitude but in direction.

Equations are next developed for the ether. Putting $\mathbf{D}$ for the electric displacement, then in ether we have div. $\mathbf{D} = 0$ where there is no electricity. Where there is, taking the integrals for a closed surface in the field, we have $\int \mathbf{D}_n d\sigma = \int \rho d\tau$, and hence

$$\int \text{div. } \mathbf{D} . d\tau = \int \rho d\tau,$$

and therefore $\qquad\qquad$ div. $\mathbf{D} = \rho$. . . . . (I.)

The charges which the ions carry about with them are supposed to occupy in each case a certain volume-space, and to gradually change in density ρ from a finite value in the interior of the space to 0 in the free ether outside. Hence, so far as the ions are concerned the electric density is different from zero only in the interior of a large number of small separate spaces. The value

of ρ at any point alters of course with the time, both on account of the movement of ions and of ponderable matter charged with electricity.

The time-rate of change of $\mathbf{D}$ or $\dot{\mathbf{D}}$ is the so-called displacement current of components $\dot{\mathbf{D}}_x$, &c. (Maxwell's $\dot{f}, \dot{g}, \dot{h}$). The total current in the dielectric is the vector sum $\rho\mathbf{v} + \dot{\mathbf{D}}$; that is, the convection-current measured by $\rho\mathbf{v}$, where $\mathbf{v}$ is the velocity of the moving matter and the displacement current. It is supposed that the solenoidal condition is fulfilled so that if $\mathbf{S}$ is the total current we have div. $\mathbf{S} = 0$. This involves the assumption that

$$\partial\mathbf{v}_x/\partial x + \partial\mathbf{v}_y/\partial y + \partial\mathbf{v}_z/\partial z = 0 ;$$

that is, that an element of volume occupied by the charge does not tend to change in amount with the displacement.

The equations of magnetic force $\mathbf{H}$ are

$$\text{div. } \mathbf{H} = 0, \quad \ldots \ldots \ldots \quad \text{(II.)}$$

$$\text{curl } \mathbf{H} = 4\pi\mathbf{S}, \quad \ldots \ldots \ldots \quad \text{(III.)}$$

and lastly there is the relation

$$-4\pi\mathrm{V}^2 \text{ curl } \mathbf{D} = \mathbf{H}. \quad \ldots \ldots \quad \text{(IV.)}$$

If there is no convection current and $\mathbf{D}$ is invariable, $\mathbf{S}$ and $\mathbf{H}$ are both zero, and we have div. $\mathbf{D} = \rho$, curl $\mathbf{D} = 0$. The latter equation is of course the condition that the components of $\mathbf{D}$ are the space-rates of variation along the axes of a function ω of x, y, z, so that $\mathbf{D}_x = -\partial\omega/\partial x/4\pi$, &c., and $\nabla^2\omega = -4\pi\rho$.

The author considers a current in a conductor as the result of the motions of innumerable charged ions, motions of course which can only be considered in a statistical manner. Thus using a bar above a quantity to denote an average value of the quantity signified, he writes for a conductor

$$\text{div. } \overline{\mathbf{H}} = 0, \quad \text{curl } \overline{\mathbf{H}} = 4\pi\overline{\mathbf{S}}, \quad \overline{\mathbf{S}} = \overline{\rho\mathbf{v}} + \overline{\dot{\mathbf{D}}}.$$

If the current system is steady, the dielectric polarisation is invariable and $\overline{\mathbf{S}} = \overline{\rho\mathbf{v}}$. If c be the volume of a sphere in which the ions move with different velocities v, the average value of ρv for the ions in the sphere is $(\int \rho\mathbf{v} \,.\, d\tau)/c$; and if the sphere is sufficiently large to give the effective average this is the mean current $\overline{\mathbf{S}}$. On the other hand, if ρ be zero except in the interior of an ion $\overline{\mathbf{S}} = \Sigma ev/c$, where e is the charge of an ion moving with velocity v. We easily obtain, if γ be the current per unit of area across an element of section of a conductor of cross $d\sigma$, $\gamma d\sigma = \Sigma ev$.

Upon a current element there is a force (electromagnetic) expressed by the vector product $[\gamma d\sigma \,.\, \mathbf{H}] = \Sigma\{e[\mathbf{v} \,.\, \mathbf{H}]\}$. The force on an ion is thus $e[\mathbf{v} \,.\, \mathbf{H}]$, and on a volume element of ponderable matter $\rho\tau[\mathbf{v} \,.\, \mathbf{H}]$.

The whole force (electric and electromagnetic) per unit of charge on the moving element is thus

$$\mathbf{F} = 4\pi V^2 \mathbf{D} + [\mathbf{v} \cdot \mathbf{H}]. \quad . \quad . \quad . \quad . \quad . \quad \text{(V.)}$$

The rate at which work is done on an element $\rho d\tau$ is thus

$$4\pi V^2 \rho d\tau (\mathbf{v}_x \mathbf{D}_x + \mathbf{v}_y \mathbf{D}_y + \mathbf{v}_z \mathbf{D}_z) ;$$

and integrating throughout the whole field or wherever there is ponderable matter we get for the total rate of working,

$$4\pi V^2 \int \rho (\mathbf{v} \ \mathbf{D}_x + \mathbf{v}_y \mathbf{D}_y + \mathbf{v}_z \mathbf{D}_z) d\tau.$$

The part $[\mathbf{v} \cdot \mathbf{H}]$ of the force does not contribute anything to the activity since it is at right angles to the direction of motion. Now we know that $4\pi (\rho \mathbf{v}_x + \dot{\mathbf{D}}_x) = [\text{curl } \mathbf{H}]_x$. Hence partial integration gives for the activity the value $dU/dt + V^2 \int [\dot{\mathbf{D}} \cdot \mathbf{H}]_n d\sigma$, where U is the energy in the field, and has the value

$$2\pi V^2 \int \mathbf{D}^2 d\tau + (\int \mathbf{H}^2 d\tau)/8\pi.$$

If there is work done by mechanical forces the activity is $d(L+U)/dt$, where L is the ordinary mechanical energy.

Localising this energy there is in unit of volume the quantity $2\pi V^2 \mathbf{D}^2 + \mathbf{H}^2/8\pi$, and the expression for the activity shows that the rate of flow of energy from within to without across the element of surface is $V^2 \int [\mathbf{D} \cdot \mathbf{H}]_n d\sigma$. This, of course, is Poynting's theorem.

Next the subject of the stresses in the ether is dealt with. Calculating the total force exerted on ponderable matter by integrating the expression for $\mathbf{F}$ given above, it is found that

$$\int \mathbf{F}_x d\tau = 4\pi V \int \mathbf{D}_x \rho d\tau + \int \rho [\mathbf{v} \cdot \mathbf{H}]_x d\tau,$$

and by using I., II., and IV., and integrating partially throughout and over a closed surface enclosing the matter, putting a, β, γ for the direction cosines of the normal to the surface element $d\sigma$,

$$\int \mathbf{F}_x d\tau = 2\pi V^2 \int \left\{ 2\mathbf{D}_x \mathbf{D}_n - a\mathbf{D}^2 + \frac{1}{8\pi} (2\mathbf{H}_x \mathbf{H}_n - a\mathbf{H}^2) \right\} d\sigma$$
$$+ \frac{d}{dt} \int (\mathbf{H}_y \mathbf{D}_z - \mathbf{H}_z \mathbf{D}_y) d\tau,$$

with similar formulas for the other components. These expressions vanish if the region of integration is empty of ponderable matter.

If now the state of the field is steady the last term on the right vanishes, and the components of stress for the element $d\sigma$ are, taken per unit of area,

$$\mathbf{X}_n = 2\pi V^2 (2\mathbf{D}_x \mathbf{D}_n - a\mathbf{D}^2) + \frac{1}{8\pi} (2\mathbf{H}_x \mathbf{H}_n - a\mathbf{H}^2) ; \ \&c.$$

This agrees with Maxwell's system of stress (since here the magnetic force and magnetic induction are taken as the same). The components here found are taken as the components of stress in the ether across the area $d\sigma$; and this stress is supposed to give rise to the ponderomotive forces on the matter imbedded in it,

It is to be observed that if the volume integral varies with the time, the Maxwellian system of stress will not suffice for equilibrium : the same result would not be obtained by integrating over a surface including the whole system of ponderable matter, and by integrating over separate surfaces enclosing the different parts of the system. For a space in which there is no ponderable matter $\int \mathbf{F}_n d\tau$ must vanish, since there div. $\mathbf{D} = 0$, curl $\mathbf{H} = 4\pi \dot{\mathbf{D}}$, and $\rho v = \mathrm{curl}\, \mathbf{H}/4\pi$. Thus the Maxwellian stress would not produce equilibrium of this part of the ether.

The author here asks why, if we suppose that the ether does not move with ponderable matter, there should be any question of force acting upon it. The simplest plan would, he suggests, be to suppose that no volume-element of ether taken as a whole is acted on by force. This would not be in accordance with the law of equality of action and reaction, since the ether exerts force on ponderable matter ; but, so far as he can see, there is nothing which obliges this law to be accepted as a fundamental principle of universal application. Of course, if the view here indicated be adopted, the stresses in the ether must be rejected as a basis for the ponderomotive forces. The author, however, does not abandon the use of the stress theory.

It is to be noted that in periodic phenomena the variable part of X_n contributes nothing to mean values of the stresses, and is therefore not taken into account in their discussion.

Considering two systems of ions to which $\rho_1, \mathbf{v}_1, \mathbf{D}_1, \mathbf{H}_1; \rho_2, \mathbf{v}_2, \mathbf{D}_2, \mathbf{H}_2$ respectively apply, let the second at time $+t$ coincide with $\rho_1, -\mathbf{v}_1, \mathbf{D}_1, -\mathbf{H}_1$ at time $-t$. This is the same as the former system with the direction of motion reversed, so that, allowing for the difference of time $2t$, the second system is simply the first reversed. The forces exerted upon the ions through the action of the ether are the same for the second system at time $+t$ as they are for the other at time $-t$.

Thus we may conceive of two motions one of which is the image of the other with regard to a fixed plane. If $\rho_1, \mathbf{v}_1, \mathbf{D}_1, \mathbf{H}_1$ be the quantities specifying one motion, then $\rho_2 = \rho_1$, and $\mathbf{v}_2, \mathbf{D}_2, \mathbf{H}_2$ must be the images of $\mathbf{v}_1, \mathbf{D}_1, -\mathbf{H}_1$. One motion is possible if the other is.

The author now proceeds to transform the equations to a system of axes moving without rotation with the velocity $\mathbf{p}$ of some part of the ponderable matter. Points are said to be at rest or in motion according as they are at rest relatively to the moving axes or not. Thus, if $\mathbf{v}$ be the velocity relatively to these axes the true velocity is $\mathbf{p} + \mathbf{v}$, and this, to give the new equations, must be used instead of $\mathbf{v}$ in the expressions already obtained.

In the new arrangement we have evidently, if $(x), (y), (z)$ be the coordinates of a point relative to these moving axes, $\partial/\partial x = \partial$ (x), &c. ; and if we designate the new differentiation with respect to the time by $\partial/\partial t$, and the old by $(\partial/\partial t)$, we have

$$\left(\frac{\partial}{\partial t}\right)_1 = \frac{\partial}{\partial t} - \mathbf{p}_x \frac{\partial}{\partial x} - \mathbf{p}_y \frac{\partial}{\partial y} - \mathbf{p}_z \frac{\partial}{\partial z}.$$

The meanings of div. $\mathbf{A}$ and curl $\mathbf{A}$ are not affected. For the former $\dot{\mathbf{D}}_x$ we now have $(\partial/\partial t - \mathbf{p}_x \partial/\partial x - \mathbf{p}_y \partial/\partial y - \mathbf{p}_z \partial/\partial z)\mathbf{D}_x$, and we shall now denote $\partial \mathbf{D}/\partial t$ by $\dot{\mathbf{D}}_x$.

Now we have the equations

$$\text{div. } \mathbf{D} = \rho ; \quad \text{div. } \mathbf{H} = 0 ; \quad \text{curl } \mathbf{H} = 4\pi \mathbf{S} ; \quad -4\pi V^2 \text{ curl } \mathbf{D} = \left(\frac{\partial \mathbf{H}}{\partial t}\right)_1 ;$$

$$\mathbf{F} = 4\pi V^2 \mathbf{D} + [\mathbf{p} \cdot \mathbf{H}] + [\mathbf{v} \cdot \mathbf{H}] ; \quad \text{and} \quad \mathbf{S} = \rho(\mathbf{p} + \mathbf{v}) + \left(\frac{\partial \mathbf{D}}{\partial t}\right)_1 ,$$

which correspond to equations I. ... V., and the equation for $\mathbf{S}$ before given. We shall refer to the first five of the equations just written as $Ia \dots Va$. From IVa it follows that

$$\dot{\mathbf{H}} = -\text{curl}\{4\pi V^2 \mathbf{D} + [\mathbf{p} \cdot \mathbf{H}]\}.$$

From $IIIa$ and the value of $\mathbf{S}$ it follows that

$$[\text{curl } \mathbf{H}]_x = 4\pi\{\rho \mathbf{v}_x + [\text{curl } \mathbf{p} \cdot \mathbf{D}]_x + \dot{\mathbf{D}}_x\},$$

or if $\mathbf{H}' = \mathbf{H} - 4\pi[\mathbf{p} \cdot \mathbf{D}]$

$$\text{curl } \mathbf{H}' = 4\pi(\rho \mathbf{v} + \dot{\mathbf{D}}).$$

These formulæ can now be applied to electrostatics. Suppose that the ions are at rest relatively to ponderable matter. We have now from IVa,

$$-4\pi V^2 \text{ curl curl } \mathbf{D} = \left(\frac{\partial}{\partial t} \text{ curl } \mathbf{H}\right)_1 ,$$

or $\quad -4\pi V^2 \left(\frac{\partial}{\partial x} \text{ div. } \mathbf{D} - \nabla^2 \mathbf{D}_x\right) = 4\pi \left(\frac{\partial}{\partial t}\right)_1 \left\{\rho(\mathbf{p} + \mathbf{v}) + \left(\frac{\partial \mathbf{D}}{\partial t}\right)_1\right\},$

which reduces to

$$V^2 \nabla^2 \mathbf{D}_x - \left(\frac{\partial^2 \mathbf{D}_x}{\partial t^2}\right)_1 = V^2 \frac{\partial \rho}{\partial x} + \left\{\frac{\partial}{\partial t} \rho(\mathbf{p} + \mathbf{v})\right\}_1 , \quad . \quad \text{(A)}$$

with two similar formulæ. Similarly can be found

$$V^2 \nabla^2 \mathbf{H}_x - \left(\frac{\partial^2 \mathbf{H}_x}{\partial t^2}\right)_1 = 4\pi V^2 \{\text{curl } \rho(\mathbf{p} + \mathbf{v})\}_x, \text{ \&c. } . \quad . \quad \text{(B)}$$

Now if $\mathbf{D}$ and $\mathbf{H}$ are independent of the time we have

$$\left(\frac{\partial}{\partial t}\right)_1 = -\left(\mathbf{p}_x \frac{\partial}{\partial x} + \mathbf{p}_y \frac{\partial}{\partial y} + \mathbf{p}_z \frac{\partial}{\partial z}\right),$$

and if a function ω be now taken which fulfils the condition

$$\nabla^2 \omega - \frac{1}{V^2}\left(\mathbf{p}_x \frac{\partial}{\partial x} + \mathbf{p}_y \frac{\partial}{\partial y} + \mathbf{p}_z \frac{\partial}{\partial z}\right)^2 \omega = \rho,$$

then by (A)

$$\mathbf{D}_x = \frac{\partial \omega}{\partial x} - \frac{\mathbf{p}_x}{V^2}\left(\mathbf{p}_x \frac{\partial \omega}{\partial x} + \mathbf{p}_y \frac{\partial \omega}{\partial y} + \mathbf{p}_z \frac{\partial \omega}{\partial z}\right); \text{ \&c.} ; \quad . \quad \text{(A')}$$

and likewise

$$\mathbf{H}_x = 4\pi\left(\mathbf{p} \frac{\partial \omega}{\partial z} - \mathbf{p}_z \frac{\partial \omega}{\partial y}\right); \text{ \&c.} . \quad . \quad . \quad \text{(B')}$$

If now the components of electric force be as before $\mathbf{F}_x$, $\mathbf{F}_y$, $\mathbf{F}_z$, we have by Va,

$$\mathbf{F}_x = 4\pi(\mathrm{V}^2 - \mathbf{p}^2)\left(\frac{\partial\omega}{\partial x}\right), \ \&c.$$

Let it be assumed that the motion of translation of the axes is in the direction of x, then $\mathbf{p}_x = \mathbf{p}$, $\mathbf{p}_y = \mathbf{p}_z = 0$, and

$$\left(1 - \frac{\mathbf{p}^2}{\mathrm{V}^2}\right)\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} + \frac{\partial^2\omega}{\partial z^2} = \rho.$$

Now consider another system in which $\omega' = \omega\sqrt{1 - \mathbf{p}^2/\mathrm{V}^2}$, $\rho' = \rho\sqrt{1 - \mathbf{p}^2/\mathrm{V}^2}$, $x' = x/\sqrt{1 - \mathbf{p}^2/\mathrm{V}^2}$; then the equation just written becomes

$$\frac{\partial^2\omega'}{\partial x'^2} + \frac{\partial^2\omega'}{\partial y^2} + \frac{\partial^2\omega'}{\partial z^2} = \rho'.$$

The x dimensions of this system are everywhere increased in the ratio of 1 to $\sqrt{1 - \mathbf{p}^2/\mathrm{V}^2}$, and the potential ω' is produced at corresponding points with the same charges as before.

The forces in this new system are related to those in the former by the equations

$$\mathbf{F}_x = \mathbf{F}'_x; \quad \mathbf{F}_y = \sqrt{1 - \mathbf{p}^2/\mathrm{V}^2}\,\mathbf{F}'_y, \ \mathbf{F}'_z = \sqrt{1 - \mathbf{p}^2/\mathrm{V}^2}\,\mathbf{F}'_z.$$

The same relations exist between the forces on the ions since the charges are the same in the two cases; and the force components vanish together in both.

The new system may be regarded as a system at rest corresponding to the one in motion with the velocity $\mathbf{p}$. Hence we obtain important conclusions. For example, to a closed conducting surface at rest and subject to a distribution partly on itself, and partly external to it, so that there is no force in its interior, there corresponds another surface with distribution upon it and external to it in which the charges are the same, but which is in motion in the direction of the axis of x, with velocity $\mathbf{p}$, and fulfils the condition that there is zero force at all points in the interior of the surface. It follows that there is zero force at every point of the interior of any conductor in motion. For to every given case a corresponding one can be found with the same charges at rest, and fulfilling the condition.

It is to be noticed also that the forces and distributions on charged bodies in motion relatively to the ether are different from those upon them when they are at rest. The attraction and repulsion of electrified bodies ought therefore to be affected by the motion of the earth. The amount of this effect is a small quantity of the second order of smallness of $\mathbf{p}/\mathrm{V}$. Since $\mathbf{p}/\mathrm{V} = 1/10,000$ about, it is not to be expected that this difference can be detected experimentally. The question whether the effect on the magnetic force given in (B') can be detected will be discussed later.   A. Gr.

[To be continued.]

CHEMICAL PHYSICS.

264. *Transition Temperature of Crystallised Sodium Sulphate.*
J. Verschaffelt. (Zschr. phys. Chem. 15. pp. 437–456, 1894.)—The temperature of the transition of Na_2SO_4, $10H_2O$ to $Na_2SO_4 + 10 H_2O$ is investigated by a method depending on the diffusion of water from a saturated solution into amyl alcohol at different temperatures. The percentage of water in the amyl alcohol is estimated by observing the temperature at which the alcohol becomes cloudy, *i. e.* is saturated. The temperature of transition is found to be $32°·74$. This method is analogous to finding the vapour-pressure of the saturated solution. A second method is used, in which the crystals are allowed to effloresce into amyl alcohol, and this gives the temperature $32°·75$. This method is analogous to finding the vapour-pressure of the crystals. These numbers agree well with the results of three other methods. From these observations heats of solution may be calculated ; and the author proceeds to show from them that the molecule of water dissolved in amyl alcohol is complex. A second calculation, based on Raoult's expression for the diminution of the vapour-pressure by the solution of a substance, leads to a least complexity of $(H_2O)_4$. (Compare Abstracts 173, 174.)   S. S.

265. *Phosphorescence at Low Temperatures.* **Raoul Pictet.** (C. R. 119. pp. 527–529, 1894.)—Any of the substances usually used to illustrate phosphorescence in physical experiments will, if exposed to sunlight and at once cooled down to about $-70°$ C., give off no phosphorescence-light whatever : as the ordinary temperature is regained, phosphorescence is re-established. As a phosphorescent substance is being cooled down, the phosphorescence-light passes through an earthy yellow before being extinguished.   A. D.

266. *Different Forms of Multiple Resonance.* **V. Bjerknes.** (Wied. Ann. 54. pp. 58–63, 1895.)—Zehnder's recent experiments seem, at first glance, to compel the conclusion that the waves from a Hertzian oscillator yield a spectrum by means of a diffraction-grating. This conclusion, however, contains a fallacy. Indeed the very same that one falls into when inferring from the researches of Sarasin and De la Rive that there are as many sets of standing waves actually present as sets of nodes observable by resonators. This mistake has been made by regarding too exclusively the general analogy between luminous and electrical rays, and forgetting that whereas luminous rays are of steady intensity and observed with an " indifferent " instrument, the electrical rays emitted by a Hertzian oscillator are *quickly damped* and, in all cases where so-called multiple resonance has appeared, have been

observed by resonators, which for a special kind of excitation have a *special sensibility*. If one explores a field of interfering electrical waves with an "indifferent" instrument (*e. g.* an electrometer not forming part of a resonating circuit), it is found to present a single set of "nodes" and "loops," like the dark and bright bands in optical interference, but fading away after a few alternations owing to the damping of the waves from the oscillator. If, however, the same field is observed with a resonator, the resonator's own peculiarities assert themselves, and it fails to give an unbiassed indication of the phenomena objectively present. This is especially so if, as usually obtains, the waves from the oscillator are rapidly damped and those in the resonator very slowly damped. The author has previously dealt with this question mathematically, and shown that the indication of a resonator is a function of the period and damping both of the oscillator and of the resonator itself.   E. H. B.

267. *Osmotic Pressure of Solutions of Finite Concentration.* **T. Ewan.** (Zschr. phys. Chem. 14. pp. 409–423, 1894.)— The equation $PV = RT$ which connects the osmotic pressure of a solution with its volume and temperature is only true for very small concentrations. The author has attempted to obtain an equation which should also apply to more concentrated solutions. It is shown that the vapour-pressure of a solution (p) is connected with its freezing-point F by the following equation :—

$$\log \frac{p_0}{p} = \frac{J}{R} \left[w_0 \frac{T_0 - F}{T_0 F} + q \frac{F - T}{TF} - c \left(\frac{T_0 - F}{F} + \log \frac{F}{T_0} \right) \right.$$
$$\left. + \gamma \left(\frac{T_1(T - F)}{FT} + \log \frac{F}{T} \right) \right], \quad . \quad (1)$$

in which p_0 is the vapour-pressure of the pure solvent and p that of the solution, both at the absolute temperature T. J is the mechanical equivalent of heat, R the gas constant for a gram-molecule, w_0 the heat of fusion for a gram-molecule of the solvent at its freezing-point T_0, q the heat of dilution of the solution at T_1, and c the difference in the heat-capacities of the liquid and solid solvent for a gram-molecule. γ is a factor which expresses the change of the heat of dilution with the temperature.

The vapour-pressures of solutions of sulphuric acid were calculated by means of this equation from their freezing-points and heats of dilution, and found to be in good agreement with those actually determined. With the help of van't Hoff's equation,

$$Pv_0 = RT \log \frac{p_0}{p}, \quad . \quad . \quad . \quad . \quad (2)$$

in which P is the osmotic pressure of the solution and v_0 the change of volume which a large quantity of the solution experiences on adding to it a gram-molecule of the solvent,

the connection between osmotic pressure and freezing-point is obtained.

By integrating Kirchhoff's equation connecting the heat of dilution and vapour-pressure of a solution, and neglecting the small variation of the heat of dilution with temperature, the following equation is obtained :—

$$\log \frac{p_0}{p} = k + \frac{JM_0}{RT}\frac{\partial Q}{\partial \omega}. \quad \cdots \cdots \quad (3)$$

M_0 is the molecular weight of the solvent (in the gaseous state) and $\frac{\partial Q}{\partial \omega}$ the heat of dilution. k (the integration constant) is a function of the concentration of the solution only. Combining (3) and (2),

$$Pv_0 = RTk + JM_0\frac{\partial Q}{\partial \omega}. \quad \cdots \cdots \quad (4)$$

A consideration of the analogy which exists between the solution of a non-electrolyte and a gas or vapour (the behaviour of the latter being taken to be represented by van der Waals' equation), led to the conclusion that probably for non-electrolytes $k = \frac{1}{n}$, n being the number of molecules of the solvent to one molecule of dissolved body. It follows from this that the molecular depression of freezing-point in solutions of non-electrolytes should be greater than the normal value calculated from van't Hoff's expression $0.02\frac{T_0^{\ 2}}{W}$ when the heat of dilution is positive, and *vice versa*. Cane-sugar in concentrated aqueous solutions gives depressions of the freezing-point which are greater than the "normal" values. The heat of dilution of such solutions was found to be positive, and the magnitudes of the heat of dilution calculated from the deviations of the molecular depressions of the freezing-point from the normal value were approximately the same as those found experimentally.   AUTHOR.

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268. *Influence of Magnetic Fields and Temperature on Electric Conductivity of Bismuth.* **J. B. Henderson.** (Wied. Ann. 53. pp. 912–923, 1894.)—The spirals of bismuth wire used were those manufactured by Hartmann and Braun for the measurement of magnetic fields. They were placed in constant magnetic fields of intensities varying from zero to 39,000 c.g.s. units, and their resistances determined while the temperature was kept constant. Their resistances were also determined at different temperatures while the field-intensity was kept constant. Temperature was found to have an unexpectedly great influence on the resistance in the magnetic field, the temperature coefficient varying with the field-intensity and also with the temperature. At constant temperature the resistance increased with the field, and at 18° C.

it was found to be tripled in a field of 39,000 c.g.s. units.   The
resistance-temperature curve for a constant field was found to be
of the following general form.   It was convex to the temperature-
axis and possessed a minimum point, the minimum point occurring
at a higher temperature the higher the field.   In fields of 7200,
9600, 11,500, and 14,200 c.g.s. the temperatures of minimum
resistance were found to be about 15°, 35°, 50°, and 75° C. respec-
tively, these temperatures falling within the limits of temperature
as yet employed, viz. 0° C. and 80° C.   The points of minimum
resistance in other fields seem to fall outside these temperature
limits.   AUTHOR.

269. *Trouton's Law of Molecular Latent Heat applied to the
Fatty Alcohols.*   **M. W. Louguinine.**   (C. R. 119. pp. 645–647,
1894.)—The values for the latent heats, which had been previously
determined by the author in the case of seven alcohols of the fatty
series, are found to comply with the relation $Mr/(T+t)=$const.,
where M is molecular weight, r latent heat, and $(T+t)$ absolute
temp.   The author emphasises the importance of the fact that the
constant has different values in case of different organic groups.
His work gives for the value of the constant for the alcohols 26·34,
while, as he mentions, R. Schiff's gives for the fatty ethers 21 and
for the aromatic hydrocarbons about 20.   F. L. T.

270. *Chemometry.*   **W. Ostwald.**   (Zschr. phys. Chem. 15.
pp. 399–408, 1894.)—The author points out that the intensity of
energy in all cases is expressed by the quotient $\dfrac{\text{energy}}{\text{capacity}}$, and that
the intensity of energy in its different conditions is measured by
well-known instruments for ascertaining temperatures, pressures,
and electrical potentials.   In the case of chemical energy no
instruments corresponding to thermometers, manometers, or elec-
trometers exists, and the paper consists of an essay on an ideal
instrument named a chemometer, which should be of the same
service in measuring the energy of chemical separation as these
other instruments in ascertaining the intensity of energy in its
other forms.   He points out that as electrical energy is equal to
chemical energy, and Faraday's law shows that chemical capacity
or quantity of matter is proportional to electrical quantity, the
intensities of chemical potential and electromotive force are also
proportional, and that therefore in this sense an electrometer is a
chemometer and can be so used for all electrolytes.   From the
theory of osmotic pressure, it follows that pressure plays a similar
part to electromotive force.   As the capacity of volume energy
is proportional to chemical capacity for equal chemical quantities
(molecular weights), its numerical value is identical with it.   S. R.

271. *Graphite.* **H. Moissan.** (C. R. 120. pp. 17–18, 1895.) —All graphites prepared at a high temperature with the help of metallic solvents, swell when treated with nitric acid at 165° or 175°. The graphites resulting from the simple heating of any carbon, diamond, lampblack, &c., never show this swelling. The swelling becomes more pronounced when the metal has been exposed to the very high temperature of the electric arc of 50 volts and 2000 amperes. Such a graphite may afterwards be dried at 120° for a whole day. The evolution of gas, the probable cause of this phenomenon, will then take place in the presence of nitric acid as soon as the temperature rises above 165°. When the experiment is made in an evacuated tube, carbonic acid and nitrogen are liberated. Some of the natural graphites exhibit this phenomenon, particularly graphites which yield ferruginous ashes. Moissan has observed it in diamond earth from the Cape, whose ashes are also rich in iron. As natural graphite is a most refractory substance, Moissan concludes that it originates from a metallic gangue, at very high temperature, without greater pressure, and that the iron has been redissolved by vapours of hydrochloric acid. At sufficiently high temperature, all carbon will pass into the graphitic state. In the presence of certain iodine and other compounds, this change takes place already at lower temperature, as Berthelot has shown. H. B.

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272. *Iron Graphite.* **H. Moissan.** (C. R. 119. pp. 1245–1250, 1894.)—Carbon is dissolved by many metals—platinum, aluminium, chromium, uranium, vanadium,—and afterwards found in the graphitic state. This graphite proves the purer and the more stable, the higher the temperature has been raised. The mixture of carbon and graphite gained from a grey cast iron melting at about 1150°, treated at red heat with chlorine, then with fuming nitric acid and hydrofluoric acid, gave crystals of a slightly grey graphite of density 2·17, which burned in oxygen at 670° and was completely transformed into graphite oxide when three times treated with nitric acid and chlorate of potash. In another series of experiments, the iron was dissolved in hydrochloric acid to which a little nitric acid had been added. This treatment destroys most of the carbon. The residue, extracted with hydrofluoric acid and boiling sulphuric acid, yielded a graphite with from 80 to 85 per cent. of C, 1·3 per cent. of ashes, and from 0·15 to 0·80 per cent. of hydrogen. Soft iron, heated for 10 minutes with an excess of sugar-carbon in the electric furnace by a current of 60 volts and 2000 amperes, became quite pasty and gave fine crystals of graphite of several millimetres length, besides some very small crystals forming a felted mass of apparently condensed carbon. The graphite, purified as before by means of chlorine and fuming nitric acid, which destroys the carbon particles, contained 99·15 per cent. of C, 0·17 of ashes, and 0·28 of H, burned in oxygen, and had a specific gravity of 2·18. It proved much more difficult to oxidise ; the resulting oxide crystallised in regular hexagons, and was not green as in the previous

cases, but colourless or somewhat smoky. Iron suddenly cooled, and by these means also strongly compressed, yielded a graphite of different appearance, with rounded edges as if of incipient fusion. From an iron in which the combined carbon had been displaced by silicon, very regular crystals of density 2·20, with 98·82 per cent. of C, were obtained. This graphite, which had not been exposed to a high temperature, was easily oxidised, the edges of the crystals being first attacked; the resulting graphite oxide of greenish-yellow colour retained the form of the graphite crystals. All these different graphites contain hydrogen, besides oxygen and sometimes nitrogen. This hydrogen is not pre-existent in the original graphite, but introduced by the treatment with dilute acids. Moissan proved this by a hydrogen determination of a test piece from the electric furnace which had simply been heated *in vacuo* for ten hours. The hydrogen and oxygen of the graphite are not given off at red heat.   H. B.

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273. *Rotativity of Ions.* **P. Walden.** (Zschr. phys. Chem. 15. pp. 196–208, 1894.)—The author determines the molecular rotation of aqueous solutions of a-bromcamphosulphonic acid and the salts it forms with lithium, sodium, potassium, thallium, beryllium, magnesium, zinc, barium, and the bases morphine and quinidine. He finds that in dilute solutions of equivalent strength the molecular rotations of the free acid and its salts of inactive bases are the same. The salts of morphine and quinidine have molecular rotations which are the algebraic sum of those of the free base and acid. If part of the water of a given solution is replaced by acetone, so that the concentration remains the same but the degree of dissociation is altered, the molecular rotation is also altered. The author concludes that the optical rotation is dependent on the number of active ions present.   J. W. C.

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